CHAPTER 5

Wildland Fire Spread Models

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I. INTRODUCTION

The spread of individual wildland fires directly affects plant communities on a short timescale. This is distinct from fire regimes over many years which help to define vegetation mosaics. It is this fairly immediate impact which has prompted a great deal of fire modeling activity over many decades. Naturally the desire to predict the spread of wildland fires to aid in containment and management of resources during particular fire incidents has also been a strong motivational factor.

In this chapter, we will survey spread models and try to give the reader an understanding of the main ideas behind the modeling as well as an overview of the different approaches and the relative merits of the various fire spread models. The emphasis will be on the explanation of relevant physical principles and

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the way in which they are encapsulated into the mathematical models. In some cases, it will be possible to completely define the model; in others, only a sketch will be possible, and the reader will need to consult the references for a complete understanding.

One topic that we will not cover is the influence of firebrands on fire propagation. This is an important topic which has only been moderately researched so far (see Lee and Hellman, 1970; several USDA Forest Service Internal Research Papers by Albini and a recent ANU Ph.D. thesis still under examination) and deserves further study. Another topic of great interest is crown fires; both their initiation and their propagation. This is an active area of ongoing research (e.g., Grishin, 1984; Albini and Stocks, 1986) and was recently briefly reviewed by Albini (1996). The fire modeling that is described in the present chapter is essential background for anyone wishing to study crown fires, firebrands, etc., but these topics will not be discussed here.

II. HEAD FIRE RATE OF SPREAD (PHYSICAL PRINCIPLES AND THEIR MATHEMATICAL EMBODIMENT)

One of the first observables of a landscape fire that would occur to most people is: "How fast did it move?" By this they usually mean the speed at which the fastest section of the fireline spreads into unburnt fuels. It was considered early in this century that the principle of energy conservation, expressed mathematically, should provide the fundamental tool for modeling and predicting the rate at which the head of a landscape fire will spread. Among the earliest published papers on this topic are Fons (1946), Bruce *et al.* (1961), Emmons (1964), Emmons and Shen (1971), Anderson (1968, 1969) Hottel *et al.* (1964, 1971), McCarter and Broido (1965), McArthur (1966), Rothermel and Anderson (1966), Albini (1967), Berlad *et al.* (1971), Thomas (1967, 1971), van Wagner (1967), and Rothermel (1972).

Later studies include Albini (1976a, 1976b, 1981, 1985, 1986), Beer (1991a, 1991b, 1993, 1995), Catchpole (1987), Cekirge (1978), De Mestre *et al.* (1989), Dorrer (1984), Duarte (1997a), Frandsen and Andrews (1979), Fujii *et al.* (1980), Gill *et al.* (1995), Green *et al.* (1985, 1990), Grishin (1984), Konev and Sukhinin (1977), Larini *et al.* (1998), Pagni and Peterson (1973), Steward (1974), Telisin (1974), Weber (1989, 1990), Weber *et al.* (1995), and Williams (1982). Literature surveys of the history and the mathematical models themselves are available in papers by Catchpole and De Mestre (1986) and Weber (1991, also translated for the *Chinese Journal of Mechanics and Practice*).

For ecological aspects see Brown (1981, 1982), da Silva (1990), Gill (1981), Green (1983a, 1983b), Greig-Smith (1979), Klukas (1973), McArthur and

Cheney (1966), Minnich (1983), Noble and Slayter (1980), Omi and Kalabokidis (1998), Rothermel and Deeming (1980), and Schmidt and Wakimoto (1987). For good general introductions, see Luke and McArthur (1978) and Chandler *et al.* (1983).

Fire spread is determined by the rate at which heat is transferred from the burning matter to unburnt matter. The way to describe this mathematically is with a precise statement of the rate at which the heat energy is accumulated in unburnt matter as a balance between energy inputs and energy outputs. To do this, we begin with the energy conservation expressed in a "word equation" as

$$\left\{\begin{matrix}\text{Rate of accumulation}\\ \text{of heat energy}\end{matrix}\right\} = \left\{\begin{matrix}\text{Rate at which}\\ \text{energy flows in}\end{matrix}\right\} - \left\{\begin{matrix}\text{Rate at which}\\ \text{energy flows out}\end{matrix}\right\}$$

This is to be applied individually to small elements of the unburnt fuel, with the fuel being considered to be a collection of many such small elements. When coupled with some ignition criteria, it then allows one to determine how quickly successive elements of fuel ignite and consequently the rate of fire spread. An algebraic relation can be obtained from this approach (Williams, 1977, 1982, 1985); namely,

$$\rho V Q_{\text{ig}} = q \qquad \text{with} \qquad Q_{\text{ig}} = \int_{T_a}^{T_i} c_p \, dT$$

This can be rearranged to give (what some people refer to as the fundamental equation for) the rate of spread

$$V = \frac{q}{\rho Q_{\text{ig}}}$$

or in words

$$\left\{\begin{matrix}\text{Rate of}\\ \text{spread}\end{matrix}\right\} = \frac{\{\text{Heat flux from the active combustion}^{1}\}}{\{\text{Heat required for fuel ignition}\}}$$

Early purely thermal models (as described in Weber, 1991) embodied the idea of many small elements in a differential equation for the temperature. An example of this style of modeling can be found in the equations in De Mestre *et al.* (1989), which then needs to be solved by integration. In fact, the solution of these models usually proceeds by assuming a constant rate of spread for the fire and changing frame of reference to a coordinate "moving with the fire." It is in this new coordinate system that the differential equation can be integrated and boundary conditions applied to complete the determination of the arbitrary constants of integration and the rate of spread. The essential requirement for

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[1] Equates with Anderson and Rothermel's "Propagating Flux"; see Section III.

accurate prediction of the rate of spread is a good description of the heat transfer processes from the fire to the fuel. This is what determines the value for the parameter A in the following example. The use of books on heat transfer, such as Gebhart (1971) or Ozisik (1973), can assist in exploring all of the options for radiative, convective, and conductive heat transfer. The fire spread papers cited in this chapter have explored many of the possibilities, with limited success when comparisons are made with experimental results.

Example

$$\rho c_p \frac{\partial T}{\partial t} = Ae^{-\alpha(x-Vt)} - lS_v(T - T_a)$$

Energy Conservation Equation

$$T(x - Vt, t) = T_i$$

Ignition Interface Condition

$$T(x \rightarrow \infty, t) = T_a$$

Ambient Condition

Step 1 Introduce a moving reference frame

$$X = x - Vt:$$

Then $$\frac{\partial T}{\partial t} = -V\frac{\partial T}{\partial X}$$

Hence $$-\rho c_p V \frac{\partial T}{\partial X} = Ae^{-\alpha X} - lS_v(T - T_a)$$

and $$T(X = 0, t) = T_i$$

Step 2 Integrate this differential equation to obtain the temperature profile:

$$V\rho c_p\{T(X, t) - T_a\}$$

$$= Ae^{lS_vX/\rho c_p} \int_{\infty}^{X} e^{-\alpha X-(lS_v/\rho c_p)X}\, dX$$

$$= \frac{Ae^{-\alpha X}}{\left(\alpha + \dfrac{lS_v}{\rho c_p}\right)}$$

Step 3 Use the condition $T(X = 0, t) = T_i$ to find the speed:

$$V = \left[\frac{A - lS_v(T_i - T_a)}{\alpha \rho c_p (T_i - T_a)} \right]$$

Notes

1. The symbol V has been used for the fire spread velocity. Often R or ROS are used in the wildland fire literature because they conjure up rate of spread. Although any definition is equally appropriate in principle, certain symbols can cause confusion when moving between research fields. As an obvious example, R is often used to denote a radius in geometry problems.

2. Simple models, such as the preceding example which can be solved exactly, can provide valuable understanding of processes and the effect of variables such as ignition temperature on measurables such as spread rate. Other similar examples, such as the lag time for fire build up, can be found in the literature (Weber, 1989). They may also form the basis for more complete simulation of a fire incident as will be shown later in this chapter, and they can also be used as the input to other models such as mesoscale meteorological models.

3. The prediction for rate of spread in this model can be seen to fit precisely with the word equation for rate of spread given at the beginning of this chapter. The heat flux from the flames, moderated by heat losses, is in the numerator, and the heat of ignition is in the denominator.

4. This sort of model only considers heat transfer aspects of fire spread. It does not model the combustion process. Rather, it assumes that the radiative output from the flame can be independently determined or estimated. More sophisticated models which include the combustion process modeled by some simplified chemical kinetic scheme, such as Grishin *et al.* (1983), Larini *et al.* (1998), and Weber *et al.* (1997), require sophisticated numerical methods for their solution and are well beyond the scope of this chapter.

III. HEAD FIRE RATE OF SPREAD: AUSTRALIA

The most widely used rate of spread models in Australia are the McArthur models for grassland fires and forest fires (McArthur, 1966; Noble *et al.*, 1980). These models make no attempt to include any physical mechanisms for fire spread. Rather, they are a purely statistical description of test fires and as such can be very successful in predicting observations in circumstances similar to the test fires. The McArthur models were developed and tested in dry grassland and forest litter in South Eastern Australia during dry winter months. Initially, the results were used to make meters, a circular slide rule which required the user to select appropriate values for degree of curing, wind velocity, and so on. The meter could then be used to obtain a fire danger index and a fire spread rate. Noble *et al.* (1980) derived mathematical relationships to represent the action of the circular slide rules. For example, the equations for the Mark 4 grassland fire danger meter are

$$F = 2\exp[-23.6 + 5.01\ln C_d + 0.0281\,T_a - 0.226\,H_r^{1/2} + 0.663\,U_{10}^{1/2}]$$

$$V = 0.036\,F$$

where C_d is the degree of curing in %, T_a is the air temperature in °C, H_r is the relative humidity in %, U_{10} is the wind velocity in m/s measured at a height of 10 m, F is the fire danger index, and V is the fire spread rate in m/s.

This type of model is easy to use and has been successful when used to predict fire spread in conditions similar to those during the test fires; for example, low-intensity fuel reduction burning. However, other fuel types such as heathlands and markedly different environmental conditions are beyond the range of variables for which these models were constructed.

IV. HEAD FIRE RATE OF SPREAD: UNITED STATES

The model most widely used in the United States was named after R. C. Rothermel (1972) who provided a practical implementation of the empirical model of Frandsen (1971). This used the principle of conservation of energy to write down an equation for rate of spread, but it did not distinguish between different modes of heat transfer. Rather, laboratory experiments, covering a range of environmental and fuel variation, were used to empirically determine the propagating flux q, as a function of reaction intensity I_R:

$$q = \frac{I_R}{(192 + 7.894\sigma)}\exp[(0.792 + 3.760\sigma^{1/2})(\beta + 0.1)]$$

where σ is the fine particle surface area to volume ratio in cm^{-1} and β is the fraction of fuel bed volume occupied by solid fuel (called the packing ratio and also equivalent to one minus the void volume fraction). The reaction intensity is related to the rate of fuel mass loss per unit area, dw/dt, according to

$$I_R = -H\frac{dw}{dt}$$

where H is the heat of combustion of the fuel. By substituting into the energy balance $q = \rho Q_{ig} V$ and rearranging, it is possible to obtain an expression for the spread rate

$$V = \frac{(H\, dw/dt)}{\rho Q_{ig}(192 + 7.894\sigma)} \exp[(0.792 + 3.760\sigma^{1/2})(\beta + 0.1)]$$

This equation highlights the important fuel bed parameters of ρ, σ, and β. The empirical nature is reflected in the functional forms found through laboratory experimentation.

Wind and slope effects upon the rate of fire spread are also included in the Rothermel model by multiplying the formula for the spread rate V by

$$(1 + \phi_W = \phi_S)$$

$$\phi_W = C_W U_{10}^{B_W} \left(\frac{\rho}{\rho_0}\right)^{-E_W}$$

$$\phi_S = 5.275\, \beta^{-0.3} \tan^2 \theta$$

where θ is the angle of the slope measured from the horizontal, B_W, C_W, and E_W are parameters involving σ, and β_0 in the optimum packing ratio. There seems to be no obvious physical reasons for the functional forms used for ϕ_W and ϕ_S, or indeed for the way in which the spread rate formula is modified to accommodate ϕ_W and ϕ_S. It is essentially an empirical model which works well in many situations. A careful examination of the parameters (Beer, 1991a) showed that the exponent to which the wind speed is raised could exceed unity (see Table 1), meaning that for sufficiently strong winds, the model would predict that the fire spreads faster than the wind moves. This is not a failure of the model. Rather, it is a consequence of the inappropriate choice for parameters, required in cases where the model was never validated. The ad hoc solution has been to place upper limits on the values of ϕ_W.

TABLE 1 Representative Values for C_W, B_W, E_W

σ (cm^{-1})	B_W	C_W	E_W
285	3.38	1.46	0.03
154	2.43	2.52	0.13
69	1.57	3.97	0.34
40	1.17	4.77	0.46
4	n/a	n/a	n/a

Following Beer (1991a).

Up until the Yellowstone fires of 1988, it was considered that the Rothermel model contained sufficient variability to accommodate most wildland fire situations. However, there has been a concerted effort over the last decade to more completely understand the limitations of the existing Rothermel model and to make improvements which will ultimately result in a new version. Not withstanding these reservations, the Rothermel model is generally a great success. It has been incorporated into a complete operational management tool and, as such, forms an integral part of the BEHAVE software—available on the World Wide Web at www.fire.org.

V. HEAD FIRE RATE OF SPREAD: CANADA

The Canadian Forest Service have conducted measurements and field experiments over a 25-year period to compile the Canadian Forest Fire Behaviour Prediction System, now available in various book forms and electronically. This provides a systematic method for including vegetation, topography, and weather variables appropriate to Canadian ecosystems and assessing wildland fire behaviour potential. It consists of mathematical formulae developed empirically and it is usually presented in tabular form or through a computer interface. With any model such as this, we can expect the predictions to be quite accurate provided our study area is reasonably represented by one of the choices available.

VI. SMOLDERING

Smoldering combustion is essentially the slow exothermic consumption of fuels (particularly organic matter in the case of wildland fires) under limited oxygen (anaerobic) conditions. This occurs quite widely in the world, from the coal seams in tropical Kalimantan (Indonesia) which cause occasional surface fires (Goldammer, 1990), to peat bogs in cool temperate areas of the world.

Large litter layers, such as those found in the boreal forests of North America, can become quite dry and support smoldering combustion, particularly in the so-called duff layer (see Chapter 13 in this book for more on smoldering duff).

The prediction of smoldering rate of spread is in principle much easier than for flaming combustion as the heat transfer aspect is greatly simplified with conduction of heat, the only mechanism of significance (Ohlemiller, 1985, 1988; Jones, 1993; Drysdale, 1999).

However, the kinetics of smoldering combustion and hence the rate of release of heat under various circumstances is not well understood. This is particularly the case when there are small traces of minerals, such as in duff layers.

For this reason, smoldering models are usually based upon the simple energy balance as introduced earlier in this chapter, and a fixed temperature is chosen for the smoldering interface.

One method for estimating the rate at which heat is released in a smoldering front is to consider the length scale of the front, namely, the distance over which the temperature rises from ambient, T_a, to the smoldering temperature, T_{sm}. If this distance is called x and if we also assume that the conductive heat transfer has reached a steady state, then the flux of heat per unit cross-sectional area is

$$q = k\frac{T_{sm} - T_a}{x}$$

We can use this in our basic combustion front propagation equation to obtain the speed

$$V = \frac{\dfrac{k(T_{sm} - T_a)}{x}}{\rho c_p(T_i - T_a)} = \frac{k}{x\rho c_p} \cdot \left(\frac{T_{sm} - T_a}{T_i - T_a}\right)$$

Given that $T_{sm} \cong T_i$, for cellulosic materials the thermal diffusivity is $k/\rho c_p \cong 10^{-7}\ \mathrm{m^2\,s^{-1}}$, and $x \cong 1$ mm, we can estimate

$$V \cong 10^{-4}\,\mathrm{ms^{-1}} \qquad \text{(i.e., 0.1 mm/s)}$$

The heat flux would have been

$$q \cong 0.01 \times \frac{300}{154} = 3 \times 10^4\,\mathrm{Js^{-1}\,m^{-2}}$$

compared to the heat of ignition

$$\rho c_p(T_i - T_a) \cong 73 \times 1370 \times 300 = 3 \times 10^7\,\mathrm{Jm^{-3}}$$

To put this into a familiar context, we note that a typical cigarette has a little less than 60 mm of tobacco. Hence, it will smolder for approximately 10 minutes if left alone in suitable circumstances.

The transition from smoldering to flaming combustion has been and continues to be an active area of research (Drysdale, 1999). It has been established beyond doubt that the main factor in the transition are limits on the supply of oxygen.

VII. WHOLE FIRE MODELING—FIRE SHAPE

So far, the fire spread models have only considered head fire rate of spread. Naturally, a whole fire consists of more than merely the fastest moving front, and the whole fire has an impact upon the landscape and particularly the vegetation. As one might expect, the intensity will vary as one traverses the fire perimeter just as the rate of spread also varies. For this reason, it has long been of interest to consider fire shape models and also to investigate mechanisms or algorithms for whole fire growth.

Some of the earliest whole fire modeling was documented by Van Wagner (1969), Kourtz and O'Regan (1971) and Anderson *et al.* (1982); although Curry and Fons (1938) and Peet (1967) had considerably earlier made selected observations on fire shape. Given homogeneous fuel and weather conditions and assuming a constant moderate wind, a fire growing from a point ignition source will evolve to a shape which we would describe as "elliptic." This is not to say that the fire shape is exactly elliptical (or some other similar mathematical curve) but that a reasonable description, in terms of accuracy and relative ease of calculating significant features, is an ellipse growing with time [but see also McAlpine and Wotton (1993)]. In the next section, this whole fire model will be described in detail, but first we should make it clear that there are many different approaches to whole fire modeling, and we will survey several of these in the remainder of this chapter (including cellular automata and percolatier approaches), but this is a growing (pun intended!) area of fire research for both management training and conservation or ecological purposes.

A. ELLIPSE MODEL OF FIRE GROWTH

Following Anderson *et al.* (1982), consider first a homogeneous fuel on level terrain with no wind, so that the conditions may be summarized as being isotropic. In this case, we expect (on average) that the fire will appear as a circular front, growing in time, and we can describe this mathematically by the parametric equations

$$\begin{aligned} x &= at\cos\chi \\ y &= at\sin\chi \end{aligned} \tag{1}$$

where x and y are the coordinates in the plane of a point on the front of the fire, a is the rate of spread (uniform across the whole front in this idealised, isotropic situation), t is the time elapsed since ignition (or some other convenient temperal reference time such as the time at which the fire is first observed in which case t would increase from some nonzero initial value t_0) and χ is a parameter which can be interpreted as an angular coordinate determining the location of the front at angles between 0° and 360° from the x-axis. That these parametric equations describe a circle can be seen by rearranging Eq. (1) and using the trigonometric identity $\sin^2 \chi + \cos^2 \chi = 1$ to give a standard form for the equation of a circle in the x, y plane; namely $x^2 + y^2 = a^2t^2$. We note that this model has the radius of the circle increasing linearly with time at a rate determined by a, the rate of spread, and a, in turn, is determined by fuel type, temperature, and moisture content.

Having established the nomenclature for describing a growing circular fire in isotropic conditions, it is possible to modify the parametric equations to include the effect of a constant wind and yield an elliptical fire front. This is done by writing the coordinates x, y of any point on the fire front as

$$\begin{aligned} x &= at(f \cos \chi + g) \\ y &= at(h \sin \chi) \end{aligned} \tag{2}$$

Clearly for the no wind case, $f = h = 1$ and $g = 0$; however, a windspeed of U will change the values of f, g, and h. The dependence upon wind speed, which we may indicate by writing $f(U)$, $G(U)$, and $h(U)$, needs to be determined in some other way, either empirically by fitting to the existing fire data or possibly from physical arguments. Note that Alexander (1985) was able to determine suitable values for a, f, g, and h for particular fires. Next, we note that Eq. (2) describe an ellipse with semiaxes atf and ath, and whose center is moving (as the fire grows) in the x-direction with a speed ag. This can most easily be seen by again using the trigonometric identity $\cos^2 \chi + \sin^2 \chi = 1$ and rearranging Eq. (2) to give

$$\left(\frac{x - agt}{aft}\right)^2 + \left(\frac{y}{aht}\right)^2 = 1$$

Anderson *et al.* (1982) derived this elliptical model for a growing fire and were then able to show that it concurred with a "modified" Huygen's principle (familiar from the ray propagation theory of light). This was a useful observation for the subsequent computer implementation of this spread model but need not concern us for the immediate purposes of this chapter.

B. ELLIPTICAL SHAPE AND WIND SPEED

The parameters in the elliptical model can be combined to give the heading, flanking, and backing rate of spread as follows:

$$a(f + g), \qquad ah, \qquad a(f - g)$$

In Figure 11 of Anderson *et al.* (1982), graphs of time series for the forward rate of spread, the lateral rate of spread, and the wind speed are all shown for a grass fire conducted by the CSIRO in Australia. This and other similar plots are not conclusive in relation to the wind speed dependence of f, g, and h. However, it is encouraging to observe that variation in the ratio of forward to lateral rate of spread, which we can write as

$$\frac{a(f+g)}{ah} + \frac{f+g}{h}$$

and plot against wind speed U (as in Figure 12 of Anderson *et al.*, 1982), can be accounted for by wind speed variation alone.

C. ELLIPTICAL SHAPE AND FIRE INTENSITY

An additional benefit of the elliptical model is that it presents a useful and reasonably simple method for extending the Byram index of head fire intensity to a measure of intensity which changes around the perimeter of the fire. The index presented by Byram (1959) is written mathematically as

$$I = H\,w\,V$$

where H is the heat of combustion of fuel, w is the mass of fuel consumed per unit area, and V is the heading rate of spread of the fire. In the elliptical model of the whole fire, we would identify $V = a(f + g)$. Hence, we could write intensity at the head, flank, and rear as

$$H\,w\,a(f+g), \qquad H\,w\,a\,h, \qquad H\,w\,a(f-g)$$

Alternatively, following Catchpole *et al.* (1982), we may prefer to determine an expression for the intensity at any point on the perimeter. For this purpose, one needs to introduce the angle of the normal to any point on the perimeter of the ellipse ψ. The result of this is an intensity equation[2]

$$I = H\,w\,a(g\cos\psi + \sqrt{f^2\cos^2\psi + h^2\sin^2\psi})$$

Catchpole *et al.* (1982) then demonstrate how this intensity varies around the perimeter for two cases idealized by

a. Medium wind: $f = 2, g = 1.8, h = 1$
b. Strong wind: $f = 4, g = 3.8, h = 1$

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[2] Note that the angle ψ has value 0 at the head of the ellipse and values $\pi/2$ and π at the flank and rear, respectively. In fact, $-\pi \le \psi \le \pi$.

In addition to using the elliptical model to determine the intensity at any point on the perimeter at any moment in time, it is also possible to determine a total fire flux by integrating the intensity around the entire perimeter.

Catchpole *et al.* (1982) performed this integration for the ellipse model to find that the total fire flux is

$$2\pi H\, w\, a^2 f\, h\, t$$

This quantity is a measurement (in units of joules per second or watts) of the energy released by the total fire per unit time. Note that this quantity increases linearly with time as does the fire perimeter.

In principle, the elliptical model allows for a comprehensive analysis of the entire progression of a fire, from its ignition and will allow estimates of fire intensity to be made at all points in space and time. Using the temperature modeling in the fire plumes chapter, these fire intensity estimates could then be used to predict temperature-time exposures for vegetation and assess the ecological impact of a fire incident, with the possibility of really comparing predictions of fire impact with observed fire impact. At this stage, such a complete incident analysis has never been undertaken.

D. OTHER FIRE SPREAD ISSUES

There have been several other approaches to modeling fire spread, often motivated by a need to deal with inhomogeneous distributions of fuel and varying topography. Unlike the elliptical models, the basic ideas have come from a very local point of view of fire spread. For example, the cellular automata approach, which was first introduced by Green *et al.* (1983, 1985), considers small cells of fuel and develops a rule approach for deciding if the fire will spread to adjacent cells. This presents a very efficient and simple method for implementing into management software and provides quite reasonable fire shapes, although there has been considerable investigation into the effect the cell shape (triangular, square, hexagonal) will have on the eventual fire shape.

There have also been several probabilistic models for fire spread, which include a probability of spread from cell to cell in a given mesh. The earliest of these was by Kourtz and O'Regan (1971), using a square mesh and fuel of varying moisture content in the mesh, distributed according to a Monte Carlo method. This then required an algorithm for determining the route of the fire. This concept was also used by Catchpole *et al.* (1989), who used a Markov chain approach to allow for several different types of fuel and also by Guertin and Ball (1990). In all of these, the concepts of transition from fuel element to fuel element and also randomness are important elements. For this reason, it has become of interest to consider the percolation approach of Albinet *et al.* (1986)

(see also Stauffer and Aharony, 1992). This arose out of developments in statistical physics (De Gennes, 1976) which were directed at finding a unified approach to spread processes with a random element. Other ecological processes (Gardner *et al.*, 1987; Turner *et al.*, 1989; Reed, 1999) have also been considered as being well modeled as percolation processes. On the experimental front, Beer (1990), Beer and Enting (1990), and Duarte (1997) have conducted laboratory studies using arrays of fuel elements. They have found good agreement between the predicted exponents from percolation theory and the clusters found experimentally.

NOTATION

ROMAN LETTERS

A	radiation intensity from fire front	$\mathrm{W\,m^{-1}}$
a	rate of spread in the absence of wind	$\mathrm{m\,s^{-1}}$
B_W	wind effect parameter dimensionless	
C_d	degree of curing %	
C_W	wind effect parameter dimensionless	
c_p	specific heat at constant pressure	$\mathrm{J\,kg^{-1}\,K^{-1}}$
E_W	wind effect parameter dimensionless	
F	fire danger index	$\mathrm{m\,s^{-1}}$
f	ellipse parameter dimensionless	
g	ellipse parameter dimensionless	
H	heat of combustion of fuel	$\mathrm{J\,kg^{-1}}$
H_r	height of wind velocity measurement	m
h	ellipse parameter dimensionless	
I	index for headfire intensity	$\mathrm{W\,m^{-1}}$
I_R	reaction intensity	$\mathrm{W\,m^{-2}}$
k	thermal conductivity	$\mathrm{W\,m^{-1}\,K^{-1}}$
l	heat loss coefficient	$\mathrm{J\,K^{-1}\,m^{-2}}$
Q_{ig}	heat of ignition	$\mathrm{J\,kg^{-1}}$
q	heat transfer from active combustion	$\mathrm{W\,m^{-2}}$
S_v	whole fuel bed surface area to volume ratio	$\mathrm{m^{-1}}$
T	temperature	K

t	time	s
U	average wind velocity	$\mathrm{m\ s^{-1}}$
U_{10}	wind velocity at a height of ten meters	$\mathrm{m\ s^{-1}}$
V	rate of spread of fire front	$\mathrm{m\ s^{-1}}$
w	mass of fuel bed per unit area	$\mathrm{kg\ m^{-2}}$
x	fixed spatial coordinate	m
X	moving spatial coordinate	m
y	fixed spatial coordinate	m

GREEK LETTERS

α	radiation absorptivity	$\mathrm{m^{-1}}$
β	packing ratio dimensionless	
θ	angle from the horizontal radians	
ρ	density	$\mathrm{kg\ m^{-3}}$
σ	fine particle surface area to volume ratio	$\mathrm{cm^{-1}}$
Φ_S	slope factor dimensionless	
Φ_W	wind factor dimensionless	
χ	angular parameter radians	
ψ	angle between normal and tangent radians	

SUBSCRIPTS

a	ambient property
ig	ignition
sm	smoldering

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