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STREAMLINES AND TRAVELTIMES OF GROUNDWATER IN A TWO-LAYERED AQUIFER SYSTEM

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INTRODUCTION

In the past in the field of geohydrological calculations it was important to determine drawdowns in groundwaterheads due to extractions. Less attention was paid to the streamlines and traveltimes of the groundwater on its way through the porous medium. Nowadays for reasons of quality of groundwater it is also very important to know which way the waterparticles take and what the time is to travel from one point to another.

In order to obtain the flowscheme and the traveltimes a calculation method was developed. This method uses the analytical solutions of the groundwaterheads for groundwaterflow towards wells. This means that generally only homogeneous and isotropic porous media can be taken into account. This makes very often a rather rough schematisation necessary. However; in many cases a very good impression can be obtained of the streamlines and traveltimes of the groundwater in the aquifer.

The method has been proven to be succesfully in the calculation of protection zones around groundwaterpumping stations. Also it is possible to predict the flow of polluted groundwater in aquifers. As a last example the use of the method can be mentioned in designing the recharge and recovery wells in a two layered aquifer system.

THE VELOCITY OF GROUNDWATER IN AQUIFERS WITH PLANE STEADY FLOW

In the case of plane steady flow in an aquifer system as indicated in figure 1 the formulas for the drawdowns in the upper and lower aquifer due to the extraction Q_1 are (ref. 1):

$$s_{11} = \frac{Q_1}{2\pi k_1 H_1 (\lambda_1 - \lambda_2)} \left((\lambda_1 - \alpha_2) K_0 (\sqrt{\lambda_1} \cdot r) + (\alpha_2 - \lambda_2) K_0 (\sqrt{\lambda_2} \cdot r) \right)$$

$$s_{21} = \frac{Q_1 \cdot \alpha_2}{2\pi k_1 H_1 (\lambda_1 - \lambda_2)} \left(-K_0 (\sqrt{\lambda_1} \cdot r) + K_0 (\sqrt{\lambda_2} \cdot r) \right)$$

There is assumed that due to the extractions there is no influence on the groundwaterhead above the semi-pervious layer with the resistance C_1.

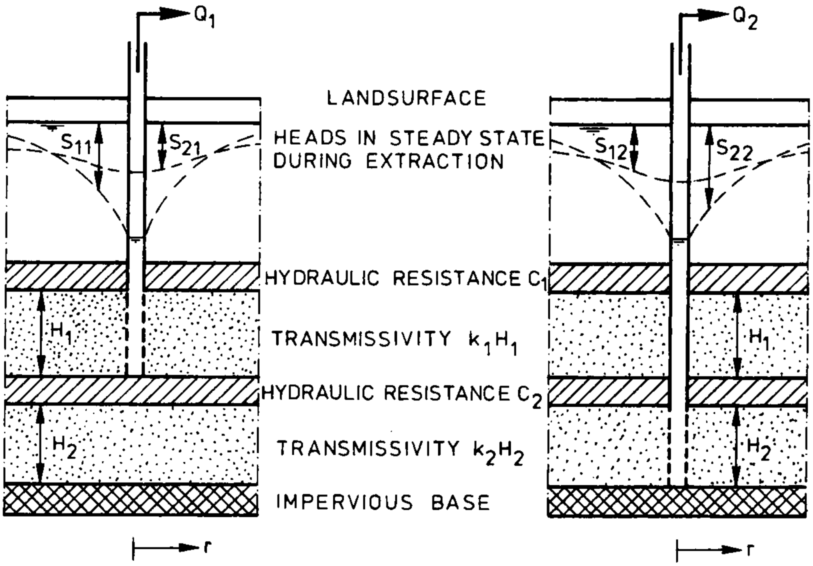

FIG. 1

Fig. 1. Drawdowns in a two layered aquifer system caused by fully penetrating wells

The drawdowns in the upper and lower aquifer due to the extraction Q_2 are (ref. 1):

$$s_{12} = \frac{Q_2 \cdot \beta_1}{2\pi k_2 H_2 (\lambda_1 - \lambda_2)} \left(-K_0\left(\sqrt{\lambda_1} \cdot r\right) + K_0\left(\sqrt{\lambda_2} \cdot r\right)\right)$$

$$s_{22} = \frac{Q_2}{2\pi k_2 H_2 (\lambda_1 - \lambda_2)} \left((\alpha_2 - \lambda_2) K_0\left(\sqrt{\lambda_1} \cdot r\right) + (\lambda_1 - \alpha_2) K_0\left(\sqrt{\lambda_2} \cdot r\right)\right)$$

$$\alpha_1 = \frac{1}{k_1 H_1 c_1} \qquad \alpha_2 = \frac{1}{k_2 H_2 c_2} \qquad \beta_1 = \frac{1}{k_1 H_1 c_2}$$

where:

s_{xy} = the drawdown in aquifer x due to an extraction in aquifer y.

$$\lambda_1 = \tfrac{1}{2}\left(\alpha_1+\alpha_2+\beta_1+\left((\alpha_1+\alpha_2+\beta_1)^2-4\alpha_1\alpha_2\right)^{1/2}\right)$$
$$\lambda_2 = \tfrac{1}{2}\left(\alpha_1+\alpha_2+\beta_1-\left((\alpha_1+\alpha_2+\beta_1)^2-4\alpha_1\alpha_2\right)^{1/2}\right)$$

r = distance to the extraction.

From Darcy's law

$$q_x = -k\cdot\frac{\partial\varphi}{\partial x} \qquad q_y = -k\cdot\frac{\partial\varphi}{\partial y}$$

it follows that, generally

$$\frac{dx}{dt} = -\frac{k}{n_e}\cdot\frac{\partial s}{\partial x}$$
$$\frac{dy}{dt} = -\frac{k}{n_e}\cdot\frac{\partial s}{\partial y}$$

where φ = groundwaterhead
s = drawdown
n_e = effective porosity

With the principle of superposition this results in a set of equations for a number of wells and a natural groundwaterflow.

For the upper aquifer these equations are:

$$\frac{dx}{dt} = \frac{-k_1}{n_1}.A.\sum_{i=1}^{m}\left[\frac{Q_i(x-x_i)}{r_i}\left((\lambda_1-\alpha_2)\sqrt{\lambda_1}.K_1(\sqrt{\lambda_1}.r_i)+(\alpha_2-\lambda_2)\sqrt{\lambda_2}K_1(\sqrt{\lambda_2}.r_i)\right)\right]+$$
$$+\frac{-k_1}{n_1}.B.\sum_{j=1}^{n}\left[\frac{Q_j(x-x_j)}{r_j}\left(-\sqrt{\lambda_1}K_1(\sqrt{\lambda_1}r_j)+\sqrt{\lambda_2}K_1(\sqrt{\lambda_2}r_j)\right)\right]+\frac{q_{nat.x}}{n_1}$$

$$\frac{dy}{dt} = \frac{-k_1}{n_1}A.\sum_{i=1}^{m}\left[\frac{Q_i(y-y_i)}{r_i}\left((\lambda_1-\alpha_2)\sqrt{\lambda_1}K_1(\sqrt{\lambda_1}r_i)+(\alpha_2-\lambda_2)\sqrt{\lambda_2}K_1(\sqrt{\lambda_2}.r_i)\right)\right]+$$
$$+\frac{-k_1}{n_1}B.\sum_{j=1}^{n}\left[\frac{Q_j(y-y_j)}{r_j}\left(-\sqrt{\lambda_1}.K_1(\sqrt{\lambda_1}.r_j)+\sqrt{\lambda_2}.K_1(\sqrt{\lambda_2}.r_j)\right)\right]+\frac{q_{nat.y}}{n_1}$$

For the lower aquifer these equations are:

$$\frac{dx}{dt} = \frac{-k_2}{n_2}.C.\sum_{i=1}^{m}\left[\frac{Q_i(x-x_i)}{r_i}\left(-\sqrt{\lambda_1}K_1(\sqrt{\lambda_1}.r_i)+\sqrt{\lambda_2}K_1(\sqrt{\lambda_2}r_i)\right)\right]+\frac{q_{nat.x}}{n_2}+$$
$$+\frac{-k_2}{n_2}.D.\sum_{j=1}^{n}\left[\frac{Q_j(x-x_j)}{r_j}\left((\alpha_2-\lambda_2)\sqrt{\lambda_1}K_1(\sqrt{\lambda_1}r_j)+(\lambda_1-\alpha_2)\sqrt{\lambda_2}K_1(\sqrt{\lambda_2}r_j)\right)\right]$$

$$\frac{dy}{dt} = \frac{-k_2}{n_2} C \cdot \sum_{i=1}^{m} \left[\frac{Q_i (y-y_i)}{r_i} \left(-\sqrt{\lambda_1} K_1 (\sqrt{\lambda_1} r_i) + \sqrt{\lambda_2} K_1 (\sqrt{\lambda_2} r_i) \right) \right] + \frac{q_{nat.y}}{n_2} +$$

$$+ \frac{-k_2}{n_2} D \cdot \sum_{j=1}^{n} \left[\frac{Q_j (y-y_j)}{r_j} \left((\alpha_2 - \lambda_2) \sqrt{\lambda_1} K_1 (\sqrt{\lambda_1} r_j) + (\lambda_1 - \alpha_2) \sqrt{\lambda_2} K_1 (\sqrt{\lambda_2} r_j) \right) \right]$$

$$r_i = \sqrt{(x-x_i)^2 + (y-y_i)^2}$$

$$r_j = \sqrt{(x-x_j)^2 + (y-y_j)^2}$$

A, B, C and D are constants depending on the hydraulic resistances and the transmissivities.

In the computerprogram FLOP these sets of simultaneous ordinary differential equations are solved with help of a Runge-Kutta solution.

As a result the streamlines and traveltimes of waterparticles in the aquifers are obtained. A plotting routine is in the computerprogram for drawing a map of the groundwater flowscheme. As an example the program FLOP has been used to calculate the streamlines and traveltimes in a recharge and recovery system.

In a two layered aquifer system a recharge takes place in the lower aquifer and an extraction in the upper aquifer. There is a natural groundwaterflow as indicated in figure 2. The resulting streamlines and traveltimes are indicated in figure 3.
The calculation is fullfilled for a traveltime of the waterparticles of 720 days. Of course this results only into a small part of the total scheme of the streamlines, but as an example this will do. In the figure 3. the streamlines in the upper aquifer are indicated with the full line, while the streamlines in the lower aquifer with the injection well are indicated with the dashed line.

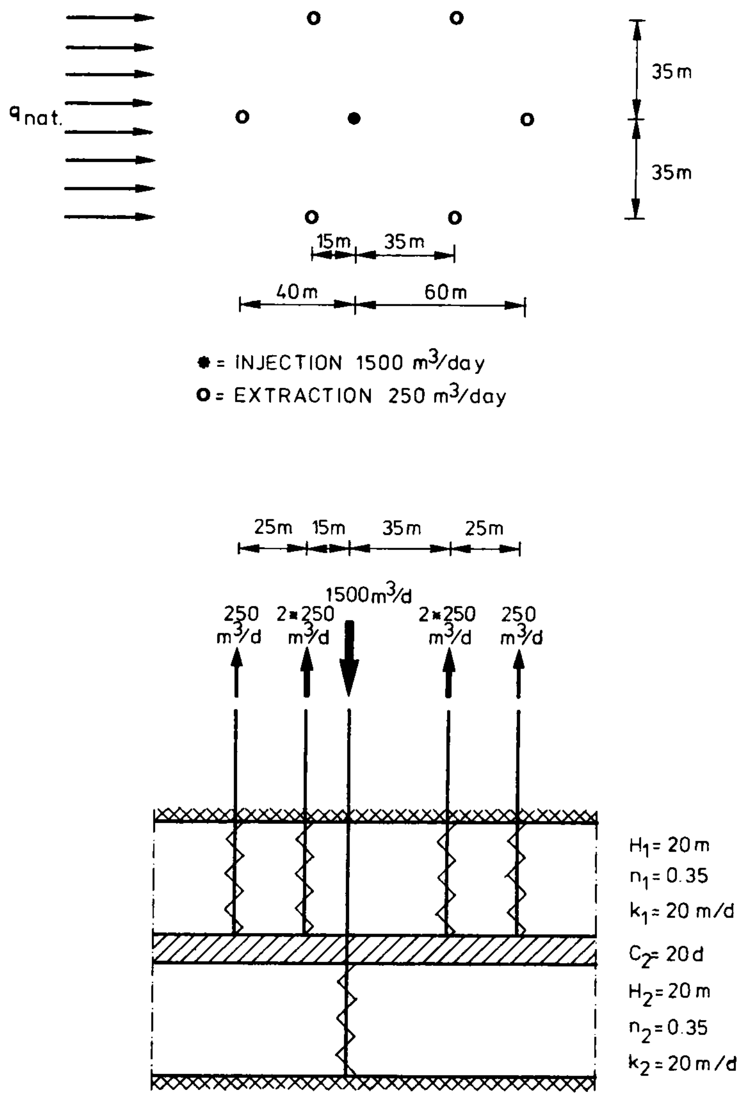
35 m
35 m
q nat.
15 m
35 m
40 m
60 m
• = INJECTION 1500 m³/day
o = EXTRACTION 250 m³/day
25 m
15 m
35 m
25 m
1500 m³/d
250 m³/d
2 * 250 m³/d
2 * 250 m³/d
250 m³/d
H1 = 20 m
n1 = 0.35
k1 = 20 m/d
C2 = 20 d
H2 = 20 m
n2 = 0.35
k2 = 20 m/d

FIG. 2

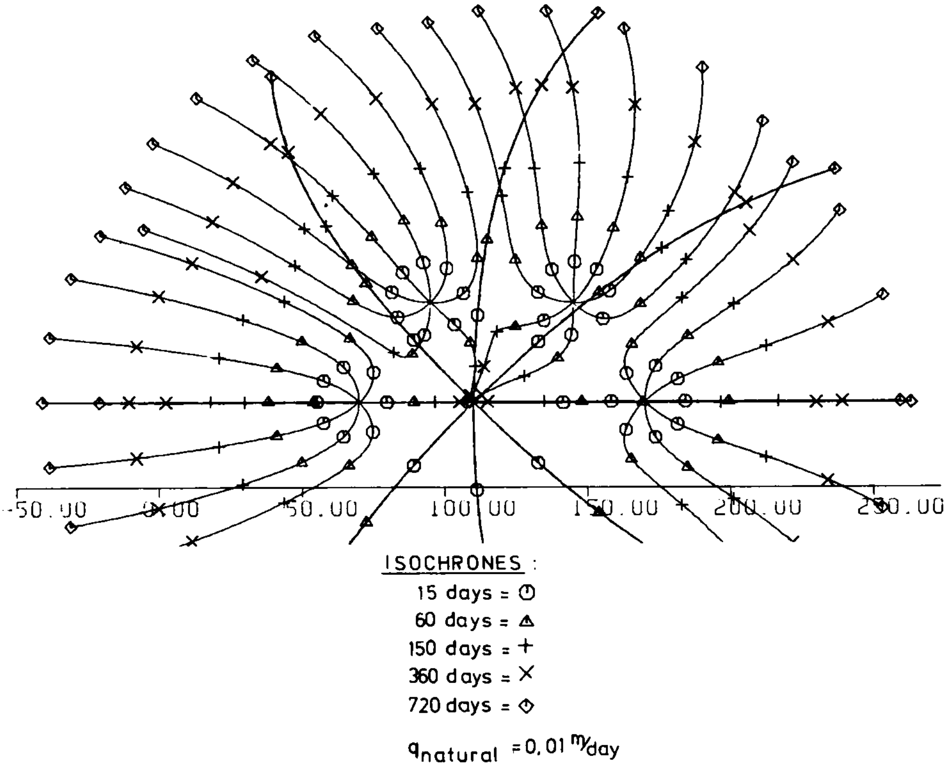

FIG.3

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