*Quality of Groundwater*, Proceedings of an International Symposium, Noordwijkerhout,
The Netherlands, 23–27 March 1981, W. van Duijvenbooden, P. Glasbergen and H. van Lelyveld (Eds.),
*Studies in Environmental Science*, Volume 17

# NUMERICAL TRACING OF THE MOVEMENT OF POLLUTANTS IN SEEPAGE BETWEEN TWO CONDUITS

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## ABSTRACT

This theoretical study concerns the tracing of pollutants in confined seepage flow between two underground conduits : one carrying polluted water, and the other carrying fresh water. Three numerical techniques are developed and compared for tracing the movement of conservative pollutants between the two conduits : linear, curvilinear, and linear-curvilinear.

The piezometric head distributions and the streamline configurations are determined numerically for steady flow through isotropic, homogeneous, porous media. Theoretical displacement fronts are then determined neglecting hydrodynamic dispersion. Comparisons between theoretical displacement fronts developed using the above three numerical techniques indicate that the linear technique is suitable generally for smaller grid sizes while the suitability of the linear-curvilinear technique extends to the large grid sizes of the meshwork enclosing the region of flow. The curvilinear technique is found to be generally suitable but requires relatively more complicated numerical procedure. Reductions of grid size decrease the differences between solutions for the displacement fronts determined using various numerical techniques and generally improve their accuracy. However, this latter tendency is diminished for excessively small grid sizes. Moreover, theoretical displacement fronts are developed using the linearcurvilinear numerical technique for flow from a surface pond to an undergound conduit and are compared to theoretical and experimental results determined by other researchers. It is shown that the theoretical results of this study agree more closely with the above experimental results than do other theoretical results.

The conclusions derived from this investigation are useful guidelines in studies concerning tracing of pollutant movement under different conditions.

## INTRODUCTION

This investigation relates, to the development of numerical techniques for salt water tracing. The problem concerns flow between two canals in a confined aquifer (see Fig. 1).

In order to verify some of the numerical techniques developed in this study comparisons are made for the problem of flow in a tile-drained field leached by ponded water (see Fig. 2). Throughout the study the porous media considered is saturated, homogeneous and isotropic. The flow is steady and laminar within the range of applicability of Darcy's law.

## ANALYSIS

In order to determine the distribution of the stream and potential function, it is necessary to solve the laplace equation numerically with the proper boundary conditions. Locations of streamlines with various values of stream functions from 10 per cent to 90 per cent (with an increment of 10 per cent) were determined through interpolation of the intersection points of the streamlines with the network in both directions (x and y). The corresponding values of the potential function at the points of intersection were determined through interpolation also. In order to calculate the intersection points of streamlines and their corresponding potential function values two interpolation methods were utilized a) linear interpolation method and b) curvilinear interpolation by Newton's divide - difference method.

Along each streamline determined previously, the accumulated time of solute advance was calculated at points of intersection with the meshwork starting from the inlet point of the streamline, located on the bed of the inlet canal proceeding along the direction of flow and terminating on the outlet point of the streamline located at the bed of the outlet canal. In order to calculate the accumulative time of solute advance along each streamline, Darcy's law may be developed for the incremental solute advance as :

$$\frac{(\Delta s)^2}{\Delta h} = \frac{K}{n} \Delta t \qquad (1)$$

where $\Delta s$, $\Delta h$, and $\Delta t$ are the incremental distance, piezometric head, and time required to move the solute on the streamline from one intersection point with the meshwork to the next; K, and n represent hydraulic conductivity and porosity respectively. In order to move the solute on the streamline through a series of intersection points with the meshwork, equation (1) becomes :

$$\sum \frac{(\Delta s)^2}{\Delta h} = \frac{K}{n} \sum \Delta t \qquad (2)$$

The left hand side of equation (2) is referred to as time factor $A_t$. Hence

$$A_t = \frac{K}{n} \sum \Delta t \qquad (3)$$

The time factor, $A_t$, is determined from the peizometric head distribution and streamline configuration. While the time required to move the solute along a streamline through a series of intersection points may be obtained from equation (3) as :

$$\sum \Delta t = \frac{(A_t)n}{K} \qquad (4)$$

The calculation of accumulated time factor, $A_t$, along each streamline was followed by an interpolation process in order to determine the locations of specified values of, $A_t$, on all streamlines. The curve through the points with the same value of $A_t$ for all streamlines represents the displacement front of the movement of solute for that time factor. Locations of specific values of $A_t$ along each streamline were determined using two types of interpolation
a) linear interpolation b) curvilinear interpolation by Newton's method.

In the modified technique the interpolation of intersection points of streamlines with the meshwork was performed using a linear relationship, whereas the locations of the selected values of accumulated time factor, $A_t$, along each streamline were determined with the aid of curvilinear interpolation. Moreover the number of points along each streamline resulting from linear interpolation of the intersection procedure were doubled by means of curvilinear interpolation method.

A computer program was written in Fortran IV in order to perform all the computation involved.

The displacement front of salt water in a two-dimensional flow system corresponding to a tile-drained field being leached by ponded water (Fig. 2) was examined in this research by using the modified numerical technique described earlier. Fernandez (ref. 3) measured displacement fronts for this case of flow in a sand tank. Luthin (ref. 5) examined it theoretically with aid of Kirkham's equations (ref. 4) for equipotentials and streamlines for the case of ponded water flow.

For this problem a drain of 0.11 ft. diameter, is located at the impervious barrier, 6 ft. from the soil surface at one end of the region of flow of length 15 ft. thus simulating a 30 ft. drain spacing. The porosity and permeability of the sand were 0.385, and 1.412 ft/hr respectively which were equal to the soil characteristics measured by Fernandez (ref. 3) in his experimental work and considered by Luthin (ref. 5) in his theoretical solution. The same problem was resolved in this investigation numerically, as indicated earlier, by using a digital computer program written in Fortran IV and the results were compared to the displacement fronts measured by Fernandez (ref. 3) and solved by Luthin (ref. 5) in order to verify the modified numerical techniques developed in this investigation.

## RESULTS

The displacement fronts at selected values of time factor, $A_t$, of 2, 4, 6, 10, 30, 50 and 70 $\ell$ for three different grid sizes of 1.667, 0.714 and 0.455 $\ell$ are shown in Figs. 3(a, b, and c) using linear, curvilinear and modified numerical techniques respectively. $\ell$ represents unit length.

The displacement fronts at selected values of time factor, $A_t$, of 2, 4, 6, 10, 30, 50 and 70 $\ell$ determined using linear, curvilinear and modified numerical technique are shown in Figs. 4(a, b, and c) for three different grid sizes of 1.667, 0.714 and 0.455 $\ell$ respectively.

Theoretical displacement fronts of salt water in a tile-draind field leached by ponded water were determined, at 4, 8, and 24 hrs. from start of drainage. Results were obtained using streamlines with various stream function values using the modified numerical technique outlined earlier. Results of these numerical solutions were compared to experimental displacement fronts measured by Fernandez (1967) and to theoretical displacement fronts calculated by Luthin (1969) as shown in Fig. 5.

## CONCLUSIONS

From the analysis of the results of this investigation, the following conclusions are drawn :

1 The modified and curvilinear techniques for the determination of

the displacement fronts yield more accurate results in comparison with the results derived from the linear technique for relatively large grid sizes in the meshwork enclosing the region of flow.

2 Differences between the solutions for the displacement fronts determined using curvilinear and modified numerical techniques are insignificant even for large grid sizes. However, the curvilinear numerical technique involves relatively more tedious numerical procedures;therefore the modified numerical technique is recommended for relatively large grid sizes in the meshwork.

3 The accuracy of each of the numerical solutions developed in this investigation for the determination of displacement front is increased as the grid size is reduced in the meshwork. Excessive reduction in the grid size do not necessarily lead to more accurate results unless adjustments are made leading to the improvement of the criterion for the termination of the interative solution of the laplace equation. Such adjustments, however, may lead to prohibitively long computer processing time.

4 Comparison of the results of the modified numerical technique with experimental measurements by Fernandez (ref. 3) for flow in a tile-drained field leached by ponded water verify the modified numerical solution and indirectly the linear and curvilinear numerical solutions.

## ACKNOWLEDGEMENTS

The authors wish to acknowledge gratefully the assistance provided by Mrs. Faiza T. Arif who typed this manuscript.

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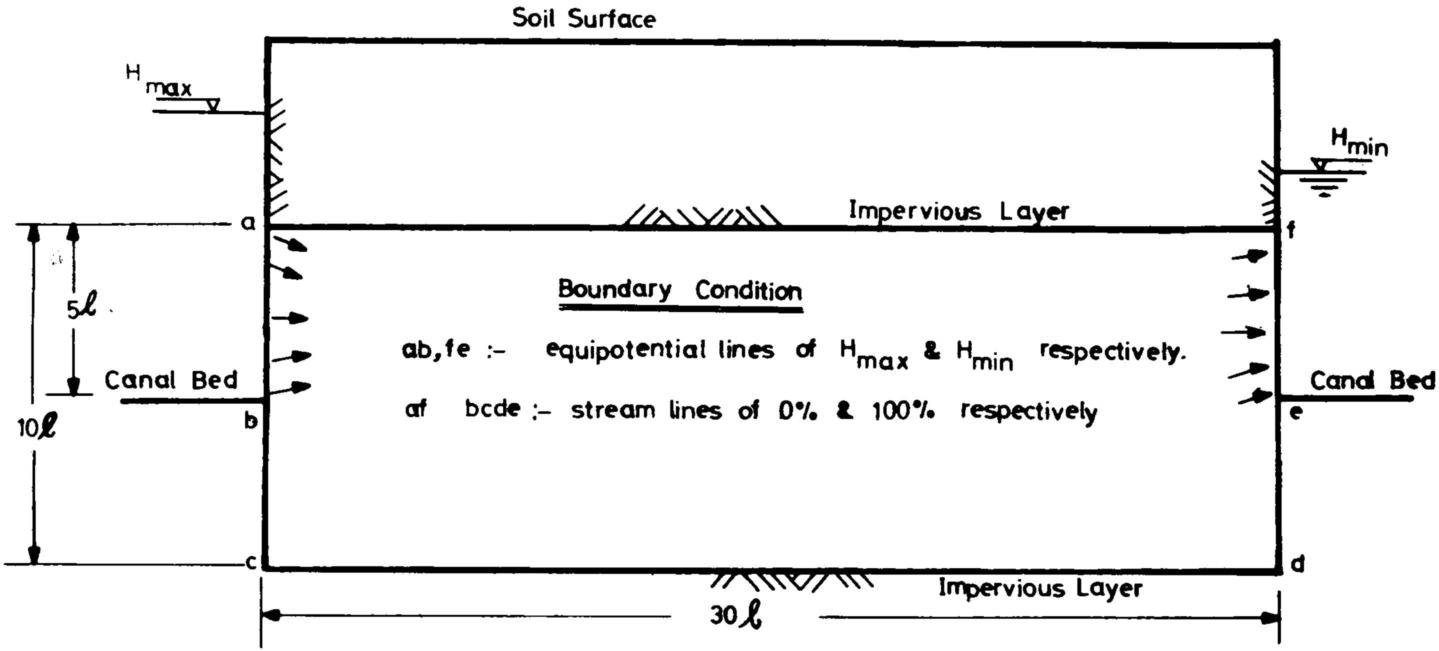

FIG(1) Flow between Two Canal in a Confined Aquifer

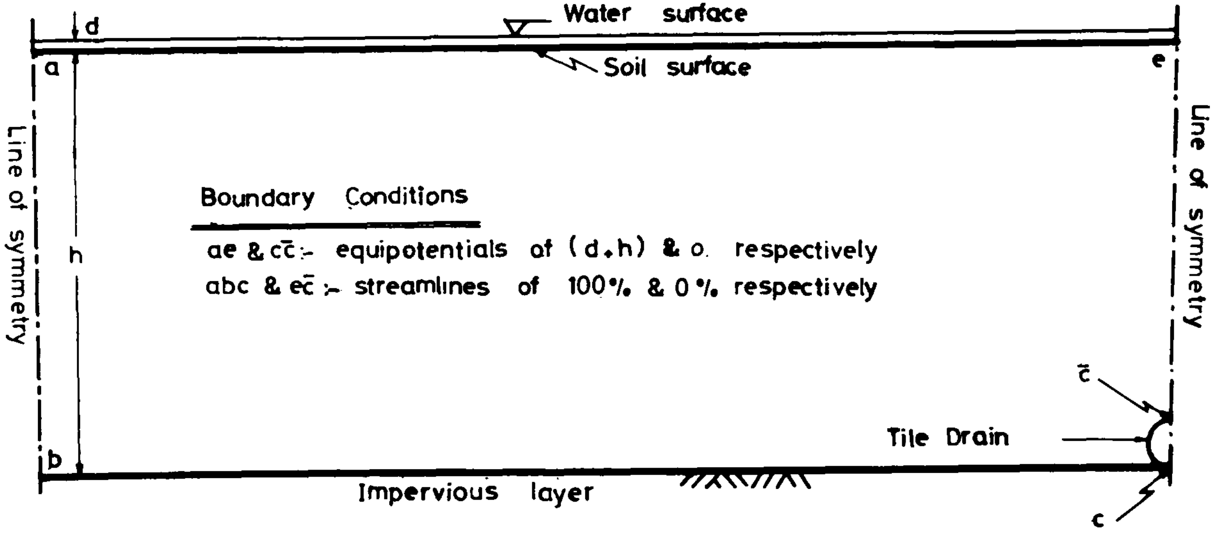

FIG.( 2) Unconfind Flow for a Tile_drained Field Leached by Ponded Water