

## SATURATED-UNSATURATED MODELLING OF GROUNDWATER QUALITY IN UNCONFINED AQUIFER SYSTEMS

Miguel A. Mariño

Department of Land, Air and Water Resources and Department of Civil Engineering,  
University of California, Davis, CA 95616 (U.S.A.)

### ABSTRACT

The transient concentration distribution of solutes within saturated-unsaturated, unconfined aquifer systems receiving infiltration is modeled with a Galerkin-type finite element method. The effects of advection, dispersion, and solute-porous media interactions which can be described by a distribution coefficient, are included. The model also includes a term for solutes which undergo first-order reactions, such as radioactive decay. The model solves for the pressure head distribution and seepage velocities in a cross-section of the flow system, and then solves for the concentration distribution of the solute. Examples that demonstrate the capability of the model are presented.

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### INTRODUCTION

Numerical simulation of groundwater quality problems has received considerable attention in the last decade. The emphasis has been in finite-element modelling of solute transport through saturated media with two-dimensional confined flow. Unsaturated flow, however, plays an important role in the movement of toxic leachates beneath landfills. Unsaturated flow conditions often prevail during the percolation of clear or polluted water from rivers, ditches, and canals toward the groundwater table. They are also encountered in dealing with watershed hydrology, unconfined aquifers, irrigation, and the drainage of agricultural lands. Analysis of many soil and groundwater contamination problems requires consideration of both the saturated and unsaturated subsurface zones. Recently, some attention has been given to finite-element modelling of water and solutes in saturated-unsaturated porous media with unconfined flow (refs. 1-5).

In this paper, a Galerkin-type finite element model is used for determining the transient movement of solutes in saturated-unsaturated, unconfined aquifer systems receiving infiltration. The model includes the effects of advection, dispersion, solute-porous media interactions which can be described by a distribution coefficient, and a source or sink for the dissolved constituent. The

model also accounts for solutes which undergo first-order reactions, such as radioactive decay. The nonlinear hydraulic properties of partially saturated porous media are represented in the model; however, hysteresis is not included. The latter may not be a serious limitation since the significance of hysteresis under field conditions is not well known at present and only infiltration (and not sequential wetting and drying) is considered in the present model. The model solves for the transient pressure head distribution and seepage velocities in a cross-section of the flow system, and then solves for the transient concentration distribution of the solute.

#### MODELLING ASPECTS

The numerical simulation of solute transport in a saturated-unsaturated subsurface system involves the solution of two partial differential equations. One differential equation is the transient flow equation that describes the flow of water in a slightly compressible unsaturated or partially saturated soil. If the pressure head distribution in the porous medium is given, the flow can be calculated by means of Darcy's law. The other differential equation is the convective-dispersion (transport) equation that describes the transient solute concentration in the system.

The flow equation is solved by a Galerkin-type finite element method (refs. 1, 2, 4, 6). To obtain continuous flow across the elements and at the nodes, a Galerkin formulation of Darcy's equation is constructed and seepage velocity vectors are calculated simultaneously at the nodes (refs. 1, 2, 4, 7). The transient velocities are used in computing hydrodynamic dispersion coefficients (ref. 8). A Galerkin-based finite element procedure is also employed to solve the convective-dispersion (transport) equation (refs. 2, 4, 7). Time integration of the flow and transport equations is carried out by fully implicit backward difference schemes (ref. 7).

The water flow portion of the model has been tested earlier (refs. 1, 2, 6). As a consequence of these tests, we have considerable confidence in the ability of the model to simulate saturated-unsaturated flow conditions. The solute transport portion of the finite element model was tested (ref. 7) against the results of analytical solutions for one-dimensional convection-dispersion in nonadsorbing (ref. 9) and adsorbing (ref. 10) porous media. The tests demonstrated the ability of the finite element model to simulate solute transport accurately, at least under the conditions specified in the tests.

#### APPLICATION TO UNCONFINED AQUIFER SYSTEMS

The Galerkin-type finite element models are used to determine the flow pattern and the distribution of solutes within unconfined aquifer systems. A schematic representation of a saturated-unsaturated, unconfined aquifer system is shown in

Fig. 1. The initial flow conditions are those of equilibrium for zero infiltration. In the flow and mass transport problems to be investigated, the following conditions are imposed: the dimensions of the porous medium are  $x_1 = 100$  ft and

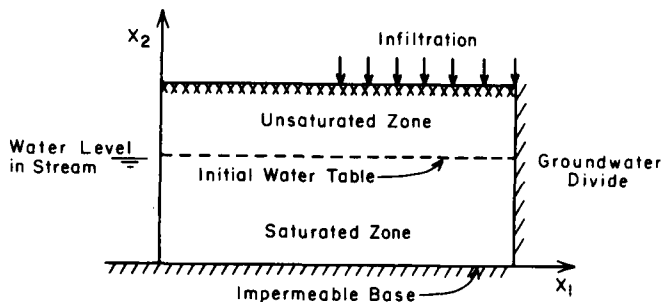


Fig. 1. Schematic representation of unconfined aquifer system receiving infiltration.

$x_2 = 60$  ft; for convenience, a constant water level of 40 ft is maintained at the left-hand side boundary; infiltration occurs at a constant and uniform rate of 2 ft/min over the right half of the top boundary; the initial pressure head distribution is -20 ft at the ground surface, 40 ft at the impermeable base, and 0 ft at the surface which separates the saturated and unsaturated portions of the porous medium; functional relationships between pressure head, relative hydraulic conductivity, and moisture content are known (Fig. 2); initially the porous medium has a relative concentration  $C/C_0 = 0.0$ ; the relative concentration of the influx

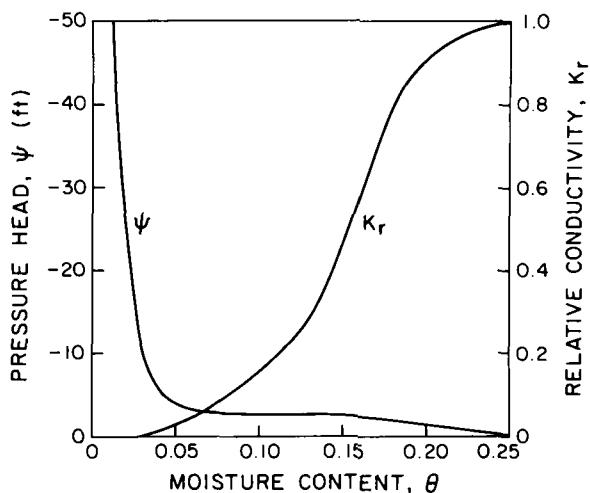


Fig. 2. Relationships between pressure head, relative hydraulic conductivity, and moisture content.

water is  $C/C_0 = 1.0$ ; and the longitudinal and transverse dispersivities of the medium ( $a_I$  and  $a_{II}$ ) are assumed to be constant over the entire domain and taken to be 20 and 5 ft, respectively, unless indicated otherwise. The finite-element discretization of the domain contains 120 elements and 77 nodes.

We consider an unconfined aquifer system in which infiltration occurs into a soil with a saturated hydraulic conductivity  $K_{11}^S = K_{22}^S = 10$  ft/min and a saturated water content  $\theta = 0.25$ . The model was run for 10 min of simulated flow time with a variable time step ranging from 0.05 to 0.5 min. Some of the results obtained, indicating the position of the  $\psi = 0$  surface, are shown in Fig. 3. The height of

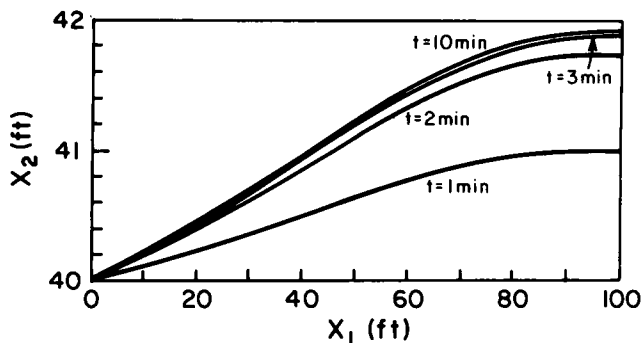


Fig. 3. Position of the  $\psi = 0$  surface at the given times.

the  $\psi = 0$  surface increases rapidly at early times and reaches almost a steady state at  $t = 3$  min. During the 10-min period the  $\psi = 0$  surface attains the maximum value of 41.9 ft. Since the position of the  $\psi = 0$  surface was obtained by interpolation between the adjacent nodes, finer grids around the  $\psi = 0$  surface were used to check the results and showed satisfactory agreement with those from the present grid. Fig. 4 shows the movement of a line of equal solute

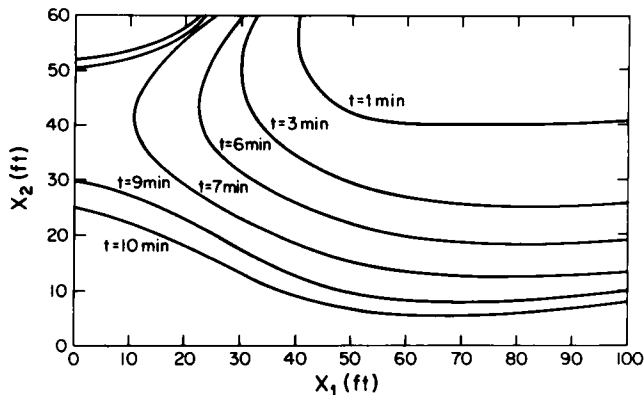


Fig. 4. Movement of a line of equal concentration ( $C/C_0 = 0.1$ ) at the given times.

concentration ( $C/C_0 = 0.1$ ) at various times. The advance speed of the line is slow in the right half of the domain where the solutes are dispersing downward; its advance speed is faster in the vicinity of the left-hand side boundary where the water flows out of the domain. The results show the effect of mechanical dispersion, whose magnitude depends largely on the seepage velocity.

Fig. 5 illustrates the effect of longitudinal and transverse dispersivities on the distribution of solutes. The values of dispersivities are varied while all other

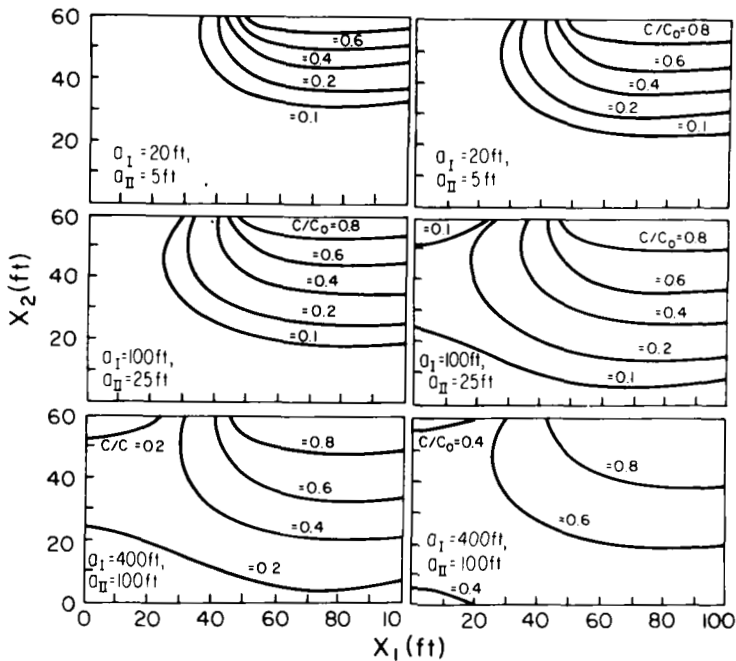


Fig. 5. Model results of relative concentration at  $t = 5$  min (left) and  $t = 10$  min (right) for saturated  $K_{11} = K_{22} = 10$  ft/min.

parameters are held constant. Three sets of dispersivities are chosen:  $a_I = 20$  ft,  $a_{II} = 5$  ft;  $a_I = 100$  ft,  $a_{II} = 25$  ft; and  $a_I = 400$  ft,  $a_{II} = 100$  ft. Note that the ratio  $a_I/a_{II}$  is the same in the three cases. The results show in general that, as the values of the dispersivities increase, the solute moves further through the flow region in a given period of time. The results also indicate that the concentration gradient decreases with an increase in the dispersivities.

Fig. 6 illustrates the effect of saturated hydraulic conductivity on the transient movement of solutes. As expected, the solutes move further through the medium as the values of saturated hydraulic conductivity increase. The results

also show that the dispersion of solute is not as sensitive to the saturated hydraulic conductivity as it is to the dispersivities.

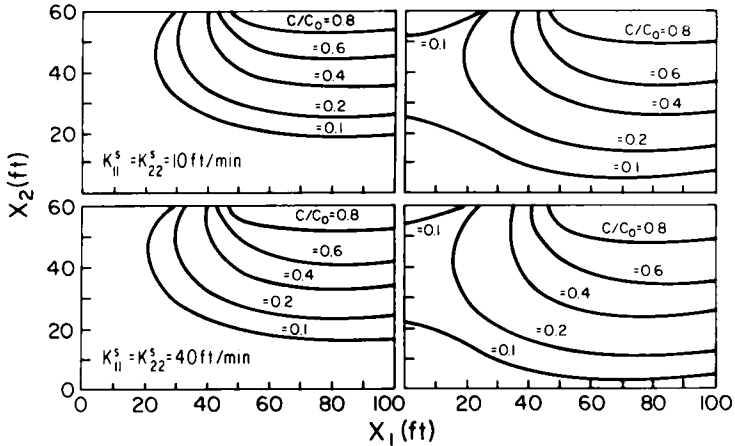


Fig. 6. Model results of relative concentration at  $t = 5$  min (left) and  $t = 10$  min (right) for  $a_I = 100$  ft,  $a_{II} = 25$  ft.

#### ACKNOWLEDGMENTS

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