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STEADY STATE MODEL OF ADVECTION AND DIFFUSION OF CONTAMINANTS IN AN INHOMOGENEOUS AQUIFER

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ABSTRACT

Dispersion of contaminants in an aquifer depends upon the direction and speed of advection, the variation of dispersion coefficient along the length of aquifer and the boundary conditions. A system of finite aquifer with uniform flow field is investigated. We have taken three kinds of boundary conditions and a simple law of variation of dispersion coefficient along the length of the aquifer. Analytical expressions for the responses of these models have been derived. These expressions have been computed for different values of Peclet number and the effects of variable dispersion coefficient and different boundary conditions on the distribution of contaminants along the length of the aquifer have been illustrated with the help of graphs.

INTRODUCTION

Fluid flow helps to localise contaminant concentration in various regions of an aquifer. Combined study of fluid flow and diffusion is attracting considerable attention because of its bearing on the water quality investigations (refs. 1-2). These studies consider mainly kinematic aspects ignoring the origin and maintenance of the flow field. Though ideally one should consider the dynamic problem determining both velocity and concentration fields (refs. 3-4), yet the kinematic studies have been found to represent significant aspects of reality. For historical development of the subject we refer to the excellent survey by Ogata (ref. 5).

Al-Niami and Rushton (ref. 1) have recently presented the steady state solutions of diffusion-advection equation for two boundary conditions. The concentration (C) was prescribed at one end of the aquifer in both cases whereas at the other end C = 0 or flux ($\equiv D \frac{dC}{dx}$) = $\frac{uC}{2}$, where u is average velocity and D, the dispersion coefficient. It is conceivable that here the total flux, instead, be zero (ref. 6). Further as aquifers are never homogeneous in reality, this assumption made by Al-Niami and Rushton (ref. 1) should also be relaxed. In this paper

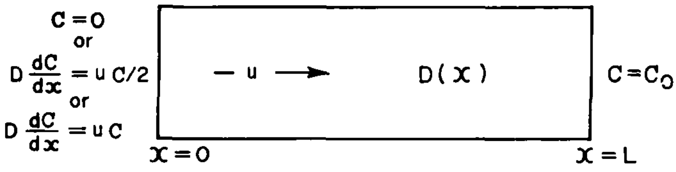

Fig. 1. Geometry of the flow system.

we present some steady state analytical solutions for three sets of boundary conditions and for a simple form of variation of D.

STEADY STATE MATHEMATICAL MODELLING

The equation for contaminant dispersion in the flow system (fig. 1) is (ref. 1)

$$\frac{d}{dx}\left(D\frac{dC}{dx}\right) - u\frac{dC}{dx} = 0 \qquad (1)$$

Here $D = D_o\, f(x)$ and u = constant. At $x = L$ (contaminant source point), we take

$$C = C_o \qquad (2)$$

At $x = 0$, if the aquifer joins a flowing fluid with zero concentration, we take,

$$C = 0 \qquad (3)$$

Another boundary condition is that the total flux is zero, i.e.,

$$D\frac{dC}{dx} - uC = 0 \qquad (4)$$

Al-Niami and Rushton (ref.1) have taken another form of the boundary condition:

$$D\frac{dC}{dx} - \frac{uC}{2} = 0 \qquad (5)$$

Here the total flux is non-zero at $x = 0$. The nature of boundary conditions was a theme of a discussion between Bear (ref. 6) and Al-Niami and Rushton (ref. 7).

The models defined by equations (1), (2) and (3); (1), (2) and (5); and (1), (2) and (4) are denoted by m1, m2 and m3 respectively. The solutions for these models are

$$C_{m1} = \{g(0,z) - 1\} / \{g(0,1) - 1\} \qquad (6)$$

$$C_{m2} = \{g(0,z) + 1\} / \{g(0,1) + 1\} \qquad (7)$$

$$C_{m3} = g(0,z) / g(0,1) \qquad (8)$$

where $z = x/L$, $C_{m1,m2,m3} = C/C_o$ (for models m1, m2 and m3, respectively), and

$$g(a,b) = \exp(uL \int_a^b D^{-1}\, dz). \qquad (9)$$

We consider two types of variation of D. For $D = D_o$, a constant,

$$g(a,b) = \exp\{Pe(b-a)\} \quad (Pe(= \frac{uL}{D_o}) \text{ is Peclet number}) \qquad (10)$$

and for exponential distribution ($D=D_o \exp(qz)$; q = constant),

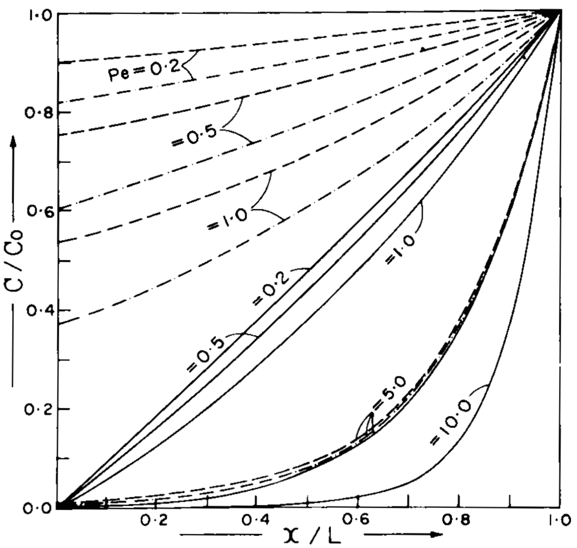

Fig. 2. Distribution of contaminants for different values of Pe for $D = D_o$

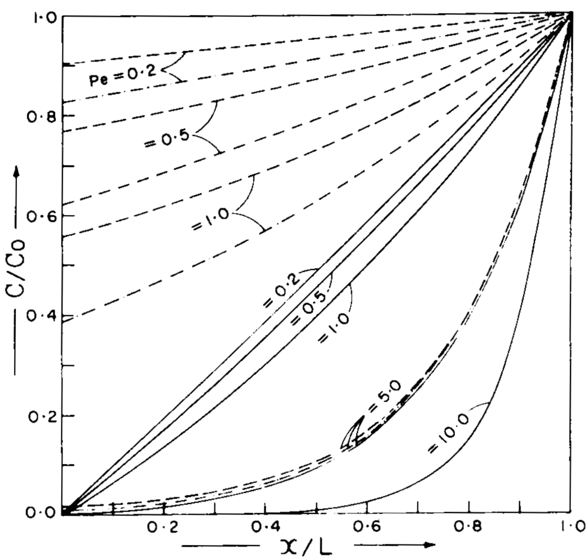

Fig. 3. Distribution of contaminants for different values of Pe for $D = D_o \exp(qz)$.

$$g(a,b) = \exp\{Pe(\exp(-qa) - \exp(-qb)) / q\} \qquad (11)$$

For determining the concentration fields for various boundary conditions, eqs. (10) and (11) are substituted in eqs. (6), (7) and (8).

COMPUTATIONAL RESULTS AND CONCLUSIONS

Figs. 2 and 3 show the distribution of contaminants for both the cases, for Peclet numbers .2,.5,1,5 and 10. The continuous lines, the broken lines, and the broken lines with dots are corresponding to the models m1, m2 and m3 respectively. Following conclusions are made from these results:

1. For smaller values of Pe, the distributions differ considerably all along the length of the aquifer while for larger values it does not differ much. For high values of Pe, the distribution of contaminants decay vary fast from the source region and the profiles come very close for these different boundary conditions.
2. For dispersivity decreasing away from the source, the distribution is significantly different than the homogeneous case. The dispersion extends upto larger distance than that in the homogeneous case. Table below gives the distances where C reduces to .1 C_o for various models.

TABLE

Distances between the source and the point where C = .1 C_o for various values of Pe.

Pe \ (L-x)/L	Homogeneous distribution			Exponential distribution		
	m1	m2	m3	m1	m2	m3
10	.225	.225	.225	.250	.250	.250
5	.445	.465	.455	.480	.510	.500
1	.840	-*	-	.855	-	-
0.5	.875	-	-	.880	-	-
0.2	.890	-	-	.900	-	-

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* $C(x) > .1C_o$ for $0 \leqslant x \leqslant L$