2 BASIC METHODS AND TOOLS FOR THE CALCULATION OF RESERVOIR FUNCTION

The methods for the analysis of reservoirs in technological parameters can be divided according to the type of hydrological data available, into those based on

- real hydrological data
- statistically processed hydrological data;

according to the procedure used into

- progressive balancing (simulation) in chronological series
- general statistical flow characteristics.

Solutions expressed in *real hydrological data* are based on direct measurements during a given observation period. However, it is presumed that the groups of flow observations compiled in the past, also sufficiently characterize the time pattern of the future exploitation of the reservoir. These groups include calculations with natural chronological flow series, as well as those with chronological series including partial corrections meant to eliminate atypical properties of the series, e.g., concerning the beginning of a wet period, etc. – although statistical methods are used for such modifications.

Calculations based on *statistically processed hydrological data* utilize hydrological data processed by probability theory methods and mathematical statistics, to obtain general statistical characteristics or synthetic hydrological pseudochronological series.

Progressive balancing (*simulation*) can be performed in real as well as in synthetic hydrological series.

The method is chosen according to the nature of the problem. Most generally valid and applicable for very complicated cases of flow regimes are methods based on modelling time chronological series (synthetic flows). Real hydrological data are used only where they can evidently assure a sufficient reliability of the results.

2.1 RESERVOIRS DESIGNED ACCORDING TO NATURAL CHRONOLOGICAL FLOW SERIES

Analysis of reservoirs according to natural chronological series is the oldest (conventional) method applied to solve any water-management problem. It was highly perfected and is still applied because of its certain inherent advantages.

In Fig. 2.1a we see the line $y = f(x)$ in a rectangular coordinate system. In Fig. way. The main disadvantage consists in the results being random according to the length and representative character of the hydrological series. It can be eliminated by a suitable choice of the design period, by evaluating how representative the series is, by modifications of the series to eliminate the anomalies and finally by statistical processing of the parameters of the series or of results.

The design principle of all water-management reservoirs, based on chronological series, consists in the successive construction of balances between the given inflow into the reservoir and the release (or withdrawal and losses) from it, dependent on or independent of the water level. The balance may be constructed numerically, resulting in a chronological series of water volumes or water levels in the reservoir with the sought characteristic and extreme values, or graphically where the result is given by a chronological curve with the same parameters.

2.1.1 Graphical methods

The advantage of graphical methods is that

- they provide a clear description of the flow control time pattern in the reservoir and thereby form an enormous aid in understanding the phenomena taking place in it,
- they are easy to survey – serious or basic mistakes become apparent at first sight,
- they permit a solution of problems which can be hardly calculated without computers,
- they permit a very prompt solution of many problems.

The accuracy of these graphical methods (with a deviation less than 2%) is quite satisfactory.

Basic tools of graphical methods

Graphical presentation use lines in plane to describe the calculated relationships. They must be arranged in such a way as to provide a quick and clear orientation. Complicated problems are graphically transformed into simpler ones needing only lines in the plane; it is also possible to combine a graphical method with a numerical one.

For a graphical presentation we have to choose

- a coordinate system,
- a scale for the most important quantities.

The coordinate system is often rectangular. Such a system is used as long as there is no special reason for abandoning it. If the coordinates reach values rendering their scale—which is limited by the size of the drawing board—inadmissably small, an oblique-angled system is chosen: the coordinates are plotted from the oblique axis (X', X'') in the direction of the Y-axis which remains vertical (see Fig. 2.1).

In Fig. 2.1a we see the line $y = f(x)$ in a rectangular coordinate system. In Fig. 2.1b we see the same line $y' = f(x)$ plotted on the same scale from the real X'-axis and line $y_1'' = f(x)$ from the real X''-axis. Finally the line y_2'' is plotted from the same X''-axis, but at twice the scale. It is sufficient to draw only the segments X'' shifted vertically by the rounded-off values (the segments) plotted on the y-axis.

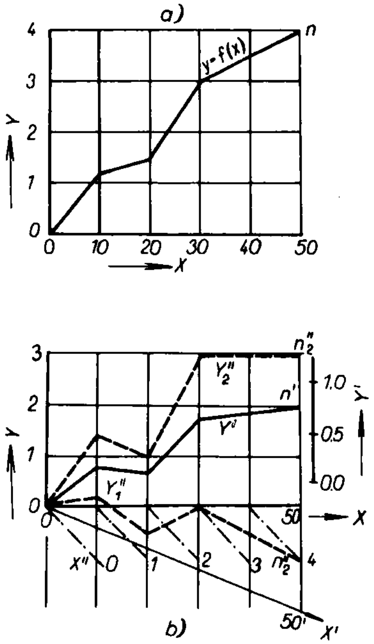

Fig. 2.1 Relationship of graphs in the rectangular and oblique coordinate systems

Figure 2.1 shows how bevelling of the X-axis results in a larger scale for the y quantities. On a rectangular raster paper (mm) the coordinate scale is uniform and not, for instance, logarithmic.

Oblique coordinates are used mainly to draw mass curves of long-term phenomena.

For graphic methods the choice of independent scales and derivations of dependent scales is of essential significance because of their great influence on accuracy and clarity.

The derived scale follows from the explicit analytical expression for the respective quantity, if on the right-hand side we substitute for all quantities units in the chosen linear scale.

If, e.g., the required quantity D is expressed analytically as

$$D\,[\mathrm{m}^3] = \frac{A\,[\mathrm{m}^3\,\mathrm{s}^{-1}] \cdot B\,[\mathrm{s}]}{C} \tag{2.1}$$

we choose the scales for the given quantities:

$A - 1\ \mathrm{m^3\,s^{-1}} = a\,[\mathrm{cm}]$
$B - 1\ \mathrm{s} = b\,[\mathrm{cm}]$
C (dimensionless number) $- 1 = c\,[\mathrm{cm}]$

and derive the scale for the requested quantity D:

$$1\ \mathrm{m^3} = d\,[\mathrm{cm}] = \frac{a\,[\mathrm{cm}] \cdot b\,[\mathrm{cm}]}{c\,[\mathrm{cm}]} \tag{2.2}$$

If the requested quantity is, e.g., the dimensionless number C, the method will be the same:

$$C = \frac{A\,[\mathrm{m^3\,s^{-1}}] \cdot B\,[\mathrm{s}]}{D\,[\mathrm{m^3}]} \tag{2.3}$$

When using the same chosen scales for A, B, D, the scale resulting for the required C will be:

$$1 = c\,[\mathrm{cm}] = \frac{a\,[\mathrm{cm}] \cdot b\,[\mathrm{cm}]}{d\,[\mathrm{cm}]} \tag{2.4}$$

Curves used in graphical presentations

Graphical methods in water management use curves which can or cannot be expressed analytically (straight lines, curves); all of which can be sub-divided into four groups:
- relation curves,
- time curves (chronological curves),
- exceedance curves,
- auxiliary curves.

All types of curves can either be given or derived. The most important derived curves are the mass curves.

The *relation curves* express the relationship between two quantities, one of which is expressed by means of the second; the relation is permanent, independent of time.

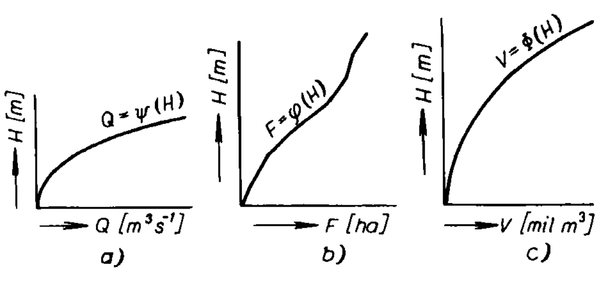

Fig. 2.2 Relation curves
(*a*) elevation–discharge; (*b*) elevation–area; (*c*) elevation–storage

Relation curves are usually applied whenever a parameter which is difficult to measure directly is to be calculated with the aid of a parameter which can be measured more easily (e.g., the discharge by measuring the water stage, Fig. 2.2a).

Time curves express the change of the quantity with time. These curves are frequently used in water management; they help to determine the variability of the flow with time (day, year, over-year period), changes in the free water level with time, etc. Parameter X therefore depends on time t, i.e., $X = F(t)$.

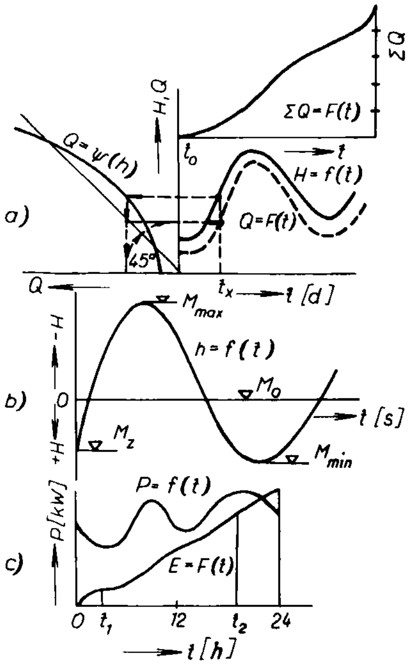

Fig. 2.3 Time pattern curves
(*a*) water stage curve and discharge curve; (*b*) fluctuation of water level curve in the compensating basin; (*c*) diagram of power system load

Figure 2.3a shows two time curves which are dependent of one another, i.e., the time course of water states in river $H = f(t)$ and the resulting time course of flow $Q = F(t)$. The two phenomena are joined by the rating curve $Q = \Psi(h)$. The construction of the points of curve Q from the points of curve H is given for time t_x in two ways and is shown by arrows.

Figure 2.3b shows the time course of the changes in the water level in the surge chamber on the conduit to the hydro-power plant for the case of a sudden interruption of release.

Figure 2.3c shows the daily course of the load of the power system $P = f(t)$.

Figure 2.3a shows also the time curve $\sum_Q = F(t)$ which is the mass curve to the flow time curve $Q = f(t)$.

In a similar way, a time curve may also express the amount of power used, E, in the electric power system by the mass curve to the daily load curve in Fig. 2.3c.

The energy consumed in the time interval $t_1 - t_2$ is given by the difference in the coordinates of the mass curve at times t_2 and t_1.

For some solutions it is sufficient to know the time interval during which a value of the phenomenon is exceeded; the phenomenon is then characterized by the so-called *exceedance curve.*

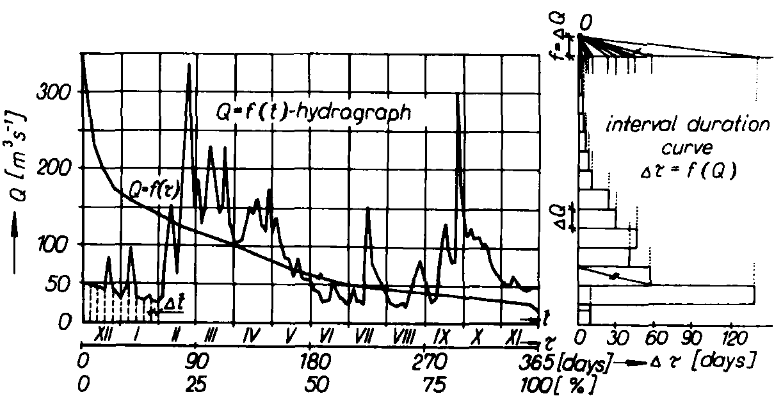

Fig. 2.4 Derivation of the discharge exceedance curve from the hydrograph (Vltava – Štěchovice, 1973)

The initial curve is the time curve. In Fig. 2.4 is the chronological curve of flow $Q = f(t)$ for the river Vltava at Štěchovice for the hydrological year 1937. From this curve we derived the exceedance curve $Q = f(\tau)$ in three ways:

(a) the flow curve is divided into vertical parts with duration Δt, these are ranked in order of magnitude, regardless of their chronological order;

(b) the flow curve is divided into horizontal parts which have heights corresponding to a flow interval ΔQ. The lengths of the corresponding time intervals are added, and their total value is horizontally plotted from the coordinate axis; this again gives us the exceedance curve which, however, should rather be denoted according to its structure $\tau = f(Q)$;

(c) the exceedance curve can also be obtained as a mass curve corresponding to the interval duration curve $\Delta t = \varphi(Q)$ derived from the chronological curve: it is divided into intervals ΔQ as in (b). The values Δt, i.e., the time of flow within the given interval, are plotted horizontally from the vertical axis as segments with the value $\Delta\tau$. The mass curve is drawn from pole o, which in our case was chosen at the pole distance $f = \Delta Q$. In this way we reach equal scales for t and τ. Thereafter, it is no longer necessary to work with pole o. A parallel to the diagonal of the respective rectangle $(\Delta\tau, \Delta Q)$ within the respective interval ΔQ can then be plotted.

If the hydrological data are given numerically, the most convenient way to draw the exceedance curve is to rank the numerical values of the parameter in order of magnitude and plot them directly as ordinates of the exceedance curve to the suitably chosen duration scale τ.

When the character of the task permits it (e.g. in the analysis of the run-of-the-river power plant), we prefer to use in the analysis the exceedance curves rather than the chronological curves, as the pattern of the former is often simpler and the analysis is less complicated.

Auxiliary curves are those which cannot be included in any of the three above-mentioned groups (e.g., the turbine efficiency curve, secants, etc.) and all auxiliary structures, helping us to reach a final graphical solution.

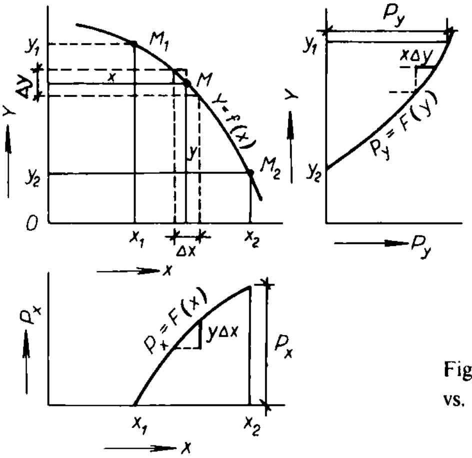

Fig. 2.5 Derivation of mass curves P_x and P_y vs. base curve $y = f(x)$

The mass curve. In water-management calculations we often have to determine the area between a curve $y = f(x)$, the x-axis and the ordinates of two points M_1 and M_2 on the curve with abscissas x_1 and x_2 (or lying inside the curve $y = f(x)$, the y-axis and the abscissa of points M_1 and M_2 with the ordinates y_1 and y_2) – Fig. 2.5.

The area base $(x_2 - x_1)$ is divided into n equal intervals Δx; the areas of the individual parts modified to rectangles $\Delta P_i = y_i \Delta x$ are calculated and then added. The volume of the entire area lying between the vertical ordinates will therefore be given by

$$P_x = y_1 \Delta x + y_2 \Delta x + \ldots + y_n \Delta x = \sum_{i=1}^{n} y_i \Delta x \tag{2.5}$$

Similarly, the area between the horizontal lines will be

$$P_y = x_1 \Delta y + x_2 \Delta y + \ldots + x_n \Delta y = \sum_{i=1}^{n} x_i \Delta y \tag{2.6}$$

The limit of the sum for $\Delta x \to 0$ (or $\Delta y \to 0$) and $n \to \infty$ is the definite integral

$$P_x = \int_{x_1}^{x_2} y \, \mathrm{d}x = \int_{x_1}^{x_2} f(x) \, \mathrm{d}x \tag{2.7}$$

or

$$P_y = \int_{y_1}^{y_2} x\,\mathrm{d}y = \int_{y_1}^{y_2} f(y)\,\mathrm{d}y \tag{2.8}$$

The integration can be performed if the function $y = f(x)$ can be expressed analytically. If that is not possible, graphical methods can be applied. To avoid working with two-dimensional planar figures, mass curves are used to reduce the problem to a single-dimensional one.

The mass curve $P_x = F(x)$ (Fig. 2.5) is obtained by successively plotting the elementary areas $y\,\Delta x$ as coordinates in the chosen scale; the finite coordinate P_x is the expression of the area x_1, x_2, M_2, M_1 in the base curve $y = f(x)$. Analogously we obtain the mass curve $P_y = F(y)$.

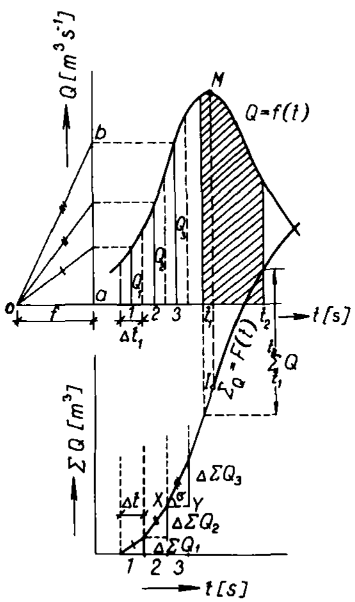

Fig. 2.6 Construction of mass discharge curve $\sum_Q$

The dimensions of the elementary areas $\Delta P_x = y\,\Delta x$ or $\Delta P_y = x\,\Delta y$ can be determined graphically. The method is shown in Fig. 2.6 – using the example of hydrograph $Q = f(t)$.

The mass hydrograph $\sum_Q$ is the mass curve corresponding to the chronological hydrograph $Q = f(t)$ and indicates by its coordinates in m^3 the water volume discharged during the chosen period t.

Construction: The area of the basic curve Q is divided into Δt wide vertical strips. Their centre lines Q_i are projected to the vertical. The pole o is chosen at a certain distance f from the vertical and from there lines are drawn through the finite points of the respective projections Q_i. In the strips Δt_i parallels to the lines are drawn and by doing so the mass curve $\sum_Q$ is obtained.

The unit value of the pole distance f = "1" follows from the scale chosen for Q, t and $\sum_Q$:

$$Q: 1\ \mathrm{m^3\,s^{-1}} = a\,[\mathrm{cm}]$$
$$t: 1\ \mathrm{s} = b\,[\mathrm{cm}]$$
$$\textstyle\sum_Q: 1\ \mathrm{m^3} = c\,[\mathrm{cm}]$$

therefore

$$f = \frac{ab}{c} = d\,[\mathrm{cm}]$$

Figure 2.7 shows the basic curve Q in wet and dry periods. A mass curve $\sum_Q$ is plotted to it from pole o and $\sum_Q'$ from pole o' which is higher, but at the same pole distance f. The basis for curve $\sum_Q$ is the axis t, for the curve $\sum_Q'$ the real basis is the axis t'.

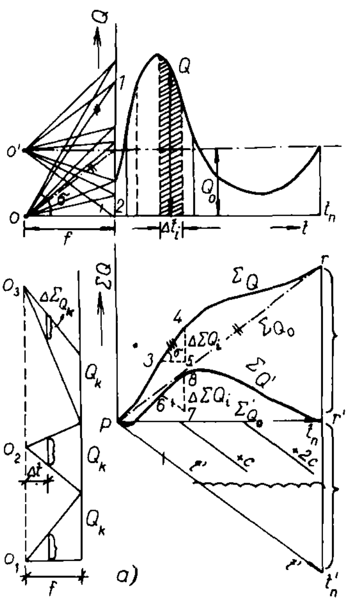

Fig. 2.7 Relationship of mass curves drawn from a raised and not raised pole with the same pole distance f

As both mass curves have a common basic curve and as the pole distances are identical, the scale of these two mass curves is also the same. That means that also the finite coordinates $\overline{t_n - r} = \overline{t_n' - r'}$ must be the same. That can be proved with the aid of similar triangles $\Delta(o\,21)$ and $\Delta(354)$ as well as $\Delta(o'\,21)$ and $\Delta(678)$, or by projecting the same Q_k from the pole o_1, o_2, o_3 at various heights but at the same distance f; in all these cases the segments of the mass curve $\Delta \sum_{Qk}$ are equal.

In order not to have to draw the real time axis of time t' to its full length, further auxiliary axes $t' + c$, $t' + 2c$, etc., are drawn plus the same constant value $+\Delta \sum_{Qk} =$ $= +c$.

The connecting line $\overline{pr}$ between the beginning and end points of the mass curve is the mass curve $\sum_Q$ for the average flow Q_o. If a parallel to $\sum_{Q_o}$ is drawn through pole o, the value Q_o is plotted on the vertical axis Q.

The same is true for the mass curve $\sum'_Q$ in relation to the higher pole. As pole o' was placed exactly at the height Q_o, the connecting line $\sum'_{Qo}$ of the beginning and end points of the mass curve $\overline{pr}$ is horizontal. The identity of the end point of the mass curve r' and point t_n on the apparent time axis proves the accuracy of the drawing.

Some characteristics of mass curves:

1. The absolute value of the basic quantity determines the slope of the tangent to the mass curve.
2. A break in the mass curve can only appear where there is a discontinuous basic function.
3. A point of inflexion on the mass curve corresponds to a relative maximum or minimum on the basic curve.
4. If the basic quantity is constant then the mass curve is a straight line.
5. At the intersection point of two base curves, the respective mass curves have parallel tangents.
6. If the pole moves on a parallel to the axis of the coordinates then the scale of the mass curves does not change. Only the direction of the real axis of the abscissae, given by the connection of the pole with the origin changes.
7. If the ordinates of the basic curve remain non-negative, the mass curve cannot decrease if the pole lies on the horizontal axis of abscissae.
8. If the value of the basic quantity is zero, the tangent of the mass curve is parallel to the connecting line between pole and origin. This rule is valid for any position of the pole and helps to define the real axis of the abscissae as a connecting line between the pole and the origin of the coordinates (or their parallel lines).
9. The parallel to the connecting line between the beginning and end points of the mass curve, drawn through the pole, gives the average value for the basic quantity on the axis of the ordinates.

Characteristics of a reservoir

The characteristics of the reservoir (Fig. 2.8) consist of two lines depicting the shape and size of the topographical configuration created by the bed and slopes of the reservoir, i.e., the depth-area curve and the curve of the reservoir volumes.

The *depth-area curve* $F = \varphi(h)$ expresses the areas of water surface corresponding to any water depth in the reservoir or to any elevation. It can be plotted from the contour line plan by determining planimetrically the areas restricted by the chosen contour line in the terrain and on the upstream face of the dam. In shallow reservoirs with intensive discharge and at the end of the backwater, hydrodynamic accumulation is more significant. The depth-area curve is not sufficient in such a case and we have to "characterize" the reservoir by cross-sections.

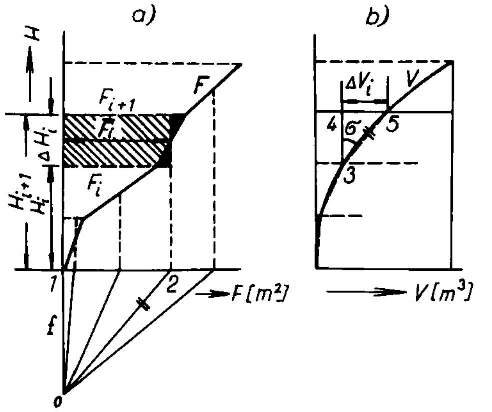

Fig. 2.8 Construction of the elevation–storage curve from the reservoir elevation–area curve

Horizontal sections at the level of the contour lines F_i divide the volume of the reservoir into ΔH_i layers. Their thicknesses correspond to the vertical distance of the contour lines. The volume of one layer is equal to

$$\Delta V_i = \frac{F_i + F_{i+1}}{2} \Delta H_i = \bar{F}_i \Delta H_i \tag{2.9}$$

where $\bar{F}_i \ldots$ is the mean size of a flooded area between two neighbouring contour lines.

The reservoir volume from the bottom to the selected level H_{n+1} is obtained as the sum of the volumes of the individual layers, i.e.

$$V = \sum_i^n \bar{F}_i \Delta H_i \tag{2.10}$$

or, if $\Delta H_i = \Delta H = \text{constant}$

$$V = \Delta H \sum_1^n \bar{F}_i \tag{2.11}$$

By adding on the coordinate axes the respective reservoir volumes to the level, we obtain the depth-volume curve $V = \Phi(H)$. Both curves, $F = \varphi(H)$ and $V = \Phi(H)$, are relation curves.

The *depth-volume curve* $V = \Phi(H)$ can be plotted as a mass curve to the depth-area curve of flooded areas $F = \varphi(H)$ (Fig. 2.8). Pole o is chosen on the prolonged axis of the ordinates at a distance f. This point is then connected with the end points of the mean coordinates $\bar{F}_i$. Parallels to these connecting lines are drawn within the boundaries of the respective layers in the reservoir $H_i \div H_{i+1}$. The distance f is that obtained by substituting units for all quantities:

$$f = \frac{\bar{F}_i \Delta H_i}{\Delta V_i} = \frac{1\,[\mathrm{m}^2] \cdot 1\,[\mathrm{m}]}{1\,[\mathrm{m}^3]} = 1 \tag{2.12}$$

The pole distance does not depend on the ΔH size, but only on its scale. If $F =$ $=$ constant, the volume curve is a straight line; if F changes according to the straight line, i.e., $F = \tan \alpha H$, the volume line, becomes a second-degree parabola, because for infinitely small distances it follows from relation (2.9)

$$\mathrm{d}V = F\,\mathrm{d}H = \tan \alpha H\,\mathrm{d}H \tag{2.13}$$

$$V = \int_0^H F\,\mathrm{d}H = \tan \alpha \int_0^H H\,\mathrm{d}H = \frac{1}{2} \tan \alpha H^2 \tag{2.14}$$

In impounding reservoirs the inundated area increases with increasing depth, which is why the deviation of the volume curve from the perpendicular increases with increasing depth. The practical result is that in a higher position in the reservoir, a larger volume ΔV increase corresponds to the same height interval ΔH.

Summation curve

The summation curve is the mass curve corresponding to the chronological line of parameter X, on which the added elementary values are represented by the product $\tau\,\Delta X$. The duration of the respective ΔX interval is expressed by τ; e.g., for the summation hydrograph

$$U_Q = \sum \tau\,\Delta Q = f(Q) \tag{2.15}$$

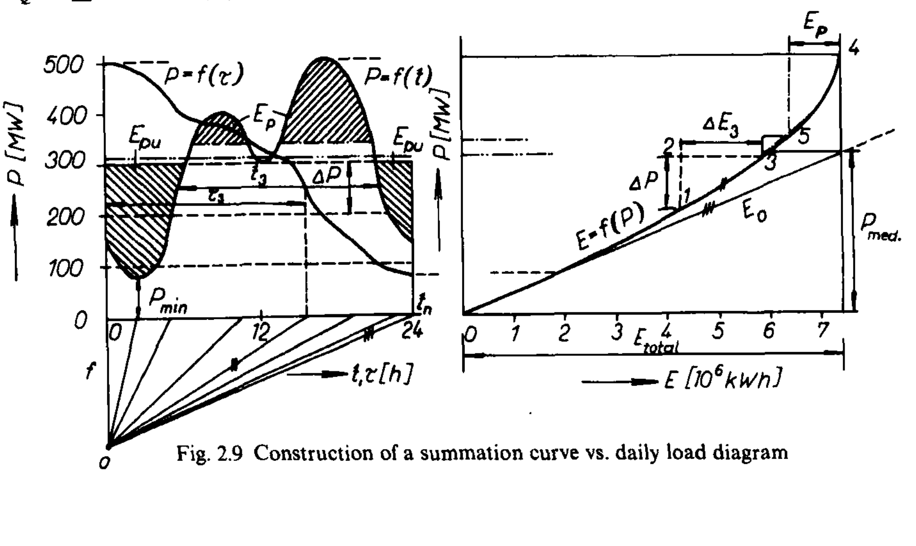

Fig. 2.9 Construction of a summation curve vs. daily load diagram

Addition in vertical direction was applied by Ježdík (1936) in his solution of compensation and distribution reservoirs. The same method was used in our derivation of the summation curve corresponding to the daily load curve when seeking the part of the load diagram which can be covered by a pumped-storage power plant.

Figure 2.9 shows the construction of a summation curve using an example from hydro-power engineering. The question is, which part of the output can be covered by a pumped storage hydro-power plant, with a power equivalent of $E_P = 1$ mil. kWh of its accumulation reservoir, in the daily load diagram. As the output of the pumped storage hydro-power plant is placed at the peak of the diagram, we have to separate the highest part of the load diagram, whose area represents 1 mil. kWh. To avoid handling the areas in the basic curve, a mass curve permitting the addition of the strips in vertical direction—i.e. the summation curve—is plotted.

Construction of the summation curve. The load diagram $P = f(t)$ is divided into horizontal strips of a constant width ΔP, the exceedance curve $\tau = f(P)$ is plotted and divided into similar strips, then the summation curve is drawn from pole o at the distance f. From similar triangles $\Delta(o0\tau_3)$ and $\Delta(123)$ it follows that

$$f = \frac{\tau \, \Delta P}{\Delta E} \tag{2.16}$$

According to the chosen scale:

$$f = 1 = \frac{1\,[\mathrm{h}] \cdot 1\,[\mathrm{kW}]}{1\,[\mathrm{kWh}]} = \frac{a \cdot b}{c}$$

Parallels to the rays from the pole trace on the strip boundaries the points through which the continuous curve $E = f(P)$ is drawn. Its coordinates E indicate the energy for the load diagram from zero to any optional output value P. The maximum coordinate E_{total} is at the level $P_{\max}$.

If we subtract from the terminal point *4* of the curve $E = f(P)$, the power equivalent of the reservoir $E_P = 1$ mil. kWh, and if we draw a vertical, this line intersects the summation curve at point 5, indicating the lower limit of the peak output which can be covered by the pumped-storage hydro-power plant. The diagram shows that the pumped-storage power plant would have to have a capacity of 160 MW, while 340 MW would be left for the other power plants in the system. The energy delivered by the pumped-storage power plant is indicated by the hatched section in this load diagram.

Summation curves can be said to have the same properties as other mass curves. Duration τ = constant means that a straight line represents the summation curve. The output P from 0 to $P_{\min}$ lasts a full 24 hours and therefore its summation curve is a straight line E_o parallel to the respective pole ray. It is easy to prove that the full load curve E_o intersects the vertical leading through the final point *4* of curve $E = f(P)$ at the mean output P_{medium}.

The horizontal distance between curves E and E_o determines the energy deficit in the given load diagram P needed for a full-load capacity of 24 hours. It is obvious that at the height P_{medium} this deficit equals the energy in the diagram above P_{medium}, which also defines the mean capacity.

One more question: "What should the minimum capacity of the other power plants in the system be if they are also to supply the energy needed for pumping the water into the accumulation reservoir of the pumped-storage power plant for a total efficiency of $\eta_c = 70\%$?" The energy demand for the pumping would amount to approximately 1.43 mil. kWh; the minimum capacity of the other power plants will be on the level at which the horizontal distance between the curves E and E_o corresponds to 1.43 mil. kWh, i.e., about 300 MW. The energy needed for pumping E_{pu} is the hatched part of the load diagram.

The properties of the summation curve in the load diagram are:

1. It indicates the respective energy for an optional output P or for the capacity scope P_1 to P_2.

2. The summation curve for full-load E_o indicates on the finite vertical line the mean output P_{medium}. A prolongation of the E_o curve to $P_{\max}$ indicates the energy to which a maximum load during 24 hours would amount, e.g., $E_{o\,\mathrm{total}} = 24P_{\max}$.

Analytical presentation of the summation curve

The continuous shape of the summation curve induced several authors to present it analytically. Comparison of the load diagram of the Czechoslovak Power System on the day of the annual maximum in 1950, with the calculations performed according to Mostkov, showed a very good coincidence. The maximum deviations amounted to 0.5–1% of the total daily energy consumption and to 2.5–6% of the peak energy (Votruba and Tvarůžek, 1960).

2.1.2 Mathematical methods

Even before computers were discovered, predetermined simple problems were solved by mathematical methods. After the introduction of computers, mathematical methods started to be applied more than graphical methods, even for complicated analytical problems for which the graphical methods had been particularly suitable.

The calculation of the function of a reservoir can be subdivided into analytical and successively balancing methods.

Analytical methods cannot be applied frequently since the functional relations are rarely able to express the basic laws of the reservoir function and its characteristics. An exact description is usually only possible in case of a simple, fully controlled inflow and withdrawal and only for reservoirs with a simple geometric shape.

The *successively balancing solution* (simulation) is usually presented in the form of

Table 2.1 Tabular calculations of reservoir storage function

Month (year 1932)	Inflow Q [$m^3 s^{-1}$]	With-drawal O_p [$m^3 s^{-1}$]	Storage $(Q - O_p)$ [$m^3 s^{-1}$]	Storage $(Q - O_p) \cdot$ 1 month [mil. m^3]	Filling and emptying of reservoir $\sum(Q - O_p) \cdot$ 1 month [mil. m^3]		Water volume in reservoir [mil. m^3] at end of month	Water volume in reservoir [mil. m^3] in middle of month
1	2	3	4	5	6	7	8	9
XI	0.90	1.10	−0.20	−0.52			1.27, 0.75	1.01
XII	1.13	1.10	+0.03	+0.08			0.83	0.79
I	0.80	1.10	−0.30	−0.78			0.05	0.44
II	1.08	1.10	−0.02	−0.05		0	0	0.02
III	2.00	1.10	+0.90	+2.33		2.33	2.33	1.17
IV	2.48	1.10	+1.38	+3.58		5.91	5.50	4.10
V	1.69	1.10	+0.59	+1.53	$\boxed{5.50}$	7.44	5.50	5.50
VI	0.79	1.10	−0.31	−0.80	4.70		4.70	5.10
VII	0.51	1.10	−0.59	−1.53	3.17		3.17	3.93
VIII	0.40	1.10	−0.70	−1.82	1.35		1.35	2.26
IX	0.58	1.10	−0.52	−1.35	0		0	0.68
X	3.02	1.10	+1.92	+4.97			4.97	2.49

1 month – number of seconds per month · 10^{-6} (average 2.63)

a table. For simple problems, calculators are quite satisfying, while complicated, extensive questions require computers.

For an identical task, a graphical and a mathematical solution require a similar mental process. Let us take the hydrological data entered in Table 2.1. and plotted

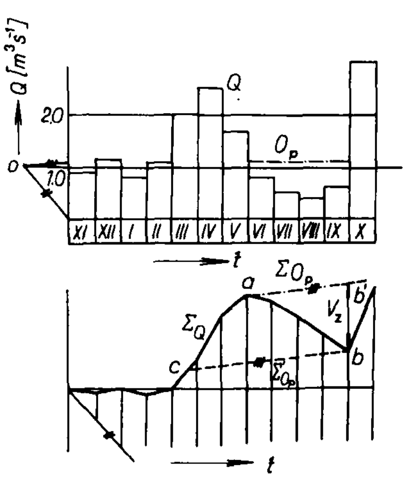

Fig. 2.10 Graphical solution of reservoir storage capacity V_z for a given yield O_p and vice-versa.

in Fig. 2.10. Let us also presume that the discharge is controlled for a constant dependable withdrawal (safe yield) O_p in a given year.

The graphical solution is then performed with the mass curves $\sum_Q$ and $\sum_{O_p}$ and for better orientation, base curves Q and O_p are also introduced.

The example can be plotted in two ways:

(a) For a given O_p we have to find the corresponding volume V_z for the active storage capacity of the reservoir. From the end of the 5th month the mass curve of the safe discharge $\sum_{O_p}$ is drawn, starting on the curve $\sum_Q$.

At the end of each month, the vertical distance between the two mass curves indicates the partial withdrawal from the active storage capacity, which we presume to be full at the beginning of the low-flow period. At the end of the 9th month the distance between the two curves is the longest; this vertical distance indicates, on the scale of the mass curves, the active storage capacity necessary to assure the constant withdrawal O_p under the given hydrological conditions. The real point of intersection C of the mass curves $\sum_Q$ and $\sum'_{O_p}$ (going through point b parallel to $\sum_{O_p}$) proves that the spring period yields enough water for the reservoir to fill up.

Similar considerations are shown in the mathematical solution in Table 2.1. We presume that the reservoir is full at the end of the 5th month and empty at the end of the 9th month. The calculation of the necessary V_z is best performed by summarizing, backwards, the multiples $(Q - O_p) \cdot$ number of seconds per month (for brevity "1 month", etc., is used) in the 5th column from the end of the low-flow period. These sums are then entered in column 6; they indicate the content of the reservoir necessary at the turn of the month to assure that it is empty at the end of the low-flow period at a given O_p. The highest value in column 6 shows the necessary volume of the active storage V_z.

A control filling of the reservoir in the spring period is performed by adding the values in column 5 from the beginning of the high-water period; the results are entered in column 7. The reservoir is full if at the moment at which it starts to discharge the value in column 7 is higher than, or at least equal to, the value in column 6.

The graphical and mathematical solutions for an over-year control are similar. In the mathematical determination of V_z we again have to calculate successively backwards from the end of the most unfavourable grouping of low-flow years.

(b) For a given V_z we have to find the achievable, dependable constant withdrawal O_p; at the end of the low-flow period (Q is no longer smaller than the O_p sought) we plot V_z vertically from point b and from the finite point b' we draw the tangent back to the $\sum_Q$ curve. In this way we get the mass curve of the dependable withdrawal $\sum_{O_p}$ and the value O_p on the parallel pole ray.

The control filling of the reservoir in the previous high-water period is performed as in (a).

The mathematical solution of a task formulated in this manner is more difficult than the graphical one, especially in a complicated hydrological series in which the periods with excess and shortage of water in the stream are not clearly separated. Here we have to use a trial and error method in which we mark on the values taken from Table 2.1 for a given $V_z = 5.5 \cdot 10^6\,\mathrm{m}^3$.

To get an idea of the yield that such an active storage capacity could offer, we transferred it to the value of a single month of yield $O_{n1} = 5.5 \cdot 10^6\,\mathrm{m}^3 : 1\ \text{month} = 2.12\,\mathrm{m}^3\,\mathrm{s}^{-1}$. The values in column 2 indicate that a yield is achieved between the 6th to 9th months. Under more complicated hydrological conditions we would not be able to determine this so definitely; we therefore have to estimate and verify the estimation afterwards.

For a dependable withdrawal we can use the natural flow in those months and the water in the storage reservoir, which we presume to be full at the beginning of the 6th month – i.e., a total volume of

$$V = (0.79 + 0.51 + 0.40 + 0.58 + 2.12) \cdot 1\ \text{month}$$

The dependable withdrawal O_p results from dividing this water volume evenly into 4 months (6th to 9th):

$$O_p = \frac{V}{4\ \text{months}} = \frac{4.40}{4} = 1.10\,\mathrm{m}^3\,\mathrm{s}^{-1}$$

The proof of having estimated the shortage period correctly is the statement that before and after this period there are months (pentads, decades, years) with a greater flow than the calculated O_p. If that were not the case, then we would also include the month with the flow $Q < O_p$ in the shortage period.

This is the answer to the main part of the question. If we wish to observe the changes in the reservoir water-stage, we have to fill in column 4, 5, 6 and 7 as in case a), but with the difference that in column 6 we need not work backwards from the end of the low-flow period, but start directly from its beginning when the reservoir is full.

Since estimation of the low-flow period is difficult, the task is often solved in the following way: several values of O_p are chosen, then we find the respective V_z and interpolation determines which O_p belongs to the given V_z.

All these mathematical and graphical methods have a serious disadvantage – they do not include the *water losses*. These are usually a function of the level in the reservoir. We therefore add more columns to our table.

9 – Water volume in the reservoir in the middle of the month ($10^6\,\mathrm{m}^3$).

10 – The respective elevations of the mid-month level in the reservoir (m above sea-level).

11 – The respective inundated areas – mid-month (hectares).

12 – Water losses related to these parameters in the middle of the respective months which are applied to the entire months ($\mathrm{m}^3\,\mathrm{s}^{-1}$).

13 – Safe net withdrawals ($\mathrm{m}^3\,\mathrm{s}^{-1}$).

With the method described above we do not arrive at the required constant safe withdrawal $O_p = 1.1\ \mathrm{m^3\,s^{-1}}$, but rather the withdrawal minus the losses and, since the losses are variable, the withdrawal is not constant. If we wish to keep to the required constant net withdrawal O_{pn}, we have to determine the course of the gross regulated withdrawal resulting after the substraction of the water losses in the required O_{pn}.

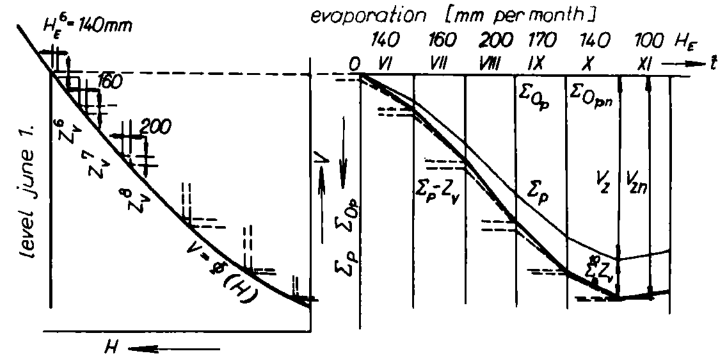

Fig. 2.11 Analysis of a storage reservoir including evaporation losses

Ježdík (1946) introduced the consideration of water losses by evaporation into the calculation of storage reservoirs. Figure 2.11 presents the graphical solution of the storage volume for reservoir V_z in mass curves with raised poles; the loss per time interval is derived from the level at the beginning of the interval, giving a slightly safer result than the reality. The basis for plotting this diagram is the reservoir volume curve V and the time behaviour of the inflow P, and of the release O_p and evaporation H_E. The mass curves $\sum_P$ and $\sum_O$ are drawn from the pole at the height O_p. Without considering the losses by evaporation, the size of the storage volume is V_z.

The *loss by evaporation* is introduced into the diagram in the following way: On the depth-volume curve we subtract the evaporation depth $H_E^6 = 140$ mm from the water stage on June 1st to find directly on the V curve the evaporation loss Z_v^6, which we subtract on the mass curve $\sum_P$ from the total inflow at the end of June. In this way we obtain the amount of water which can be used for release from the June inflow. From the water stage on July 1st we again subtract the evaporation depth $H_E^7 =$ $= 160$ mm, and so on.

If we wish to maintain the net release $O_{pn} = O_p$, the reservoir is emptied more than if the evaporation were not considered. The difference is exactly the volume of the evaporated water. We have to refill not only the release O_p from the storage volume, but also the evaporation loss Z_v. The reservoir volume must then necessarily amount to $V_{zn} > V_z$. Evaporation can be introduced into the calculation in a simpler manner if it is not an important factor in the balance.

The above-described table and graphical solution were based on the principle that a certain V_z has to be matched with the largest possible safe withdrawal O_p that is able to guarantee it during the given period, or to match a given O_p with the smallest possible active storage capacity V_z resulting in a 100% reliability for that period. As the hydrological series is limited, the real reliability is $p < 100\%$.

Using the tabular form it is possible to solve the tasks with reliability of $p < 100\%$. We achieve this by eliminating one or several years (periods) with the lowest flow.

In practice we often analyse in the given case the function

$$f(V_z, O_p, p) = 0 \tag{2.17}$$

or in relative values

$$f(\beta_z, \alpha, p) = 0 \tag{2.18}$$

Suitable for such analyses are the "relative" storage-yield curves $\beta_z = f(\alpha)$ or a statistical processing of the results of calculation in hydrological series (see Chapter 6).

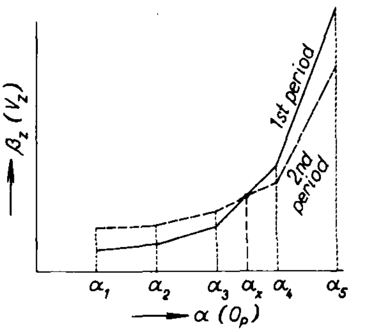

Fig. 2.12 Storage-yield curves $\beta_z = f(\alpha)$ or $V_z = f(O_p)$

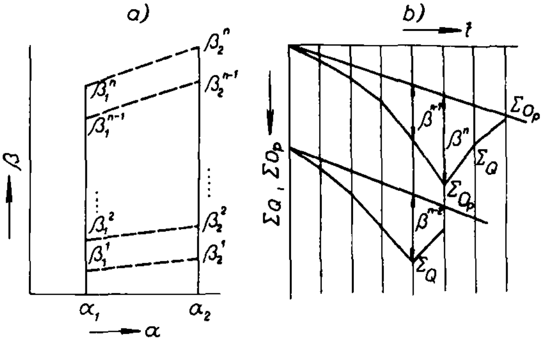

Fig. 2.13 The principle of the determination of the storage-yield curves for various probabilities p
(*a*) diagram of β_i^j values; (*b*) decisive low-flow period

The curves $\beta_z = f(\alpha)$ for a given hydrological series can, e.g., be calculated or plotted using graphical analysis for chosen values $\alpha_1, \alpha_2, \ldots$ by determining the respective values $\beta_{z1}, \beta_{z2}, \ldots$ in the individual low-flow periods (Fig. 2.12). With their aid we determine which period is decisive for the chosen α and which β_z value belongs to it (according to Fig. 2.12 the 2nd period is decisive for $\alpha < \alpha_x$ and the first period for $\alpha > \alpha_x$). In the given hydrological series the reliability $p = 100\%$ corresponds to the upper envelope of all storage-yield relation curves. The lower envelope, plotted from a larger number of relation curves, belongs to the reliability $p < 100\%$.

To arrive at reliable value for the reliability p, especially for a high $p \to 0.99$, we need a statistical processing of the results or computation using synthetic hydrological series.

The program can be compiled according to the instructions given in Fig. 2.12. For a chosen α_i we get n $\beta_i^1, \beta_i^2, \ldots, \beta_i^n$ values (Fig. 2.13a). For the requisite $p = 0.99$ we have to have $n \gtreqless 100$, or even better a multiple of n. With $n = 100$, the storage-yield relation curve for $p = 0.99$ consists of the points $\beta_1^{n-1}, \beta_2^{n-1}, \ldots, \beta_m^{n-1}$ belonging to the values α_i $(i = 1, 2, \ldots, m)$; for $p = 0.98$ these will be the points $\beta_1^{n-2}, \beta_2^{n-2}, \ldots, \beta_m^{n-2}$, etc. When determing the values β^j $(j = 1, 2, \ldots, n)$ care must be taken to determine correctly the low-flow period in which the respective β^j appears (Fig. 2.13b).

2.2 PROBABILITY AND STATISTICAL METHODS

The application of probability theory and of mathematical statistics to the solution of problems relating to flow control by reservoirs is justified by two conditions:

(a) that the problems to be solved deal with processes having random (probability, stochastic) character,

(b) that the historical observed series is only one of the infinite number of possible realizations of the random process which will never be repeated in an identical form in the future.

A *random phenomenon* is the basic concept of the probability theory, its occurrence cannot be forecast exactly in each single case, even if a certain complex of conditions is preserved. Random can be a quantity (variable) or an entire process. A *random variable* is characterized by the set of all its possible values and by its probability distribution; a *random process* is characterized by the set of all its possible time functions and the laws determining the statistical properties of this set.

In water management there are many quantities and processes with a random character and for that reason the probability theory and methods of mathematical statistics can be applied to them. They can be used for calculating the release control of reservoirs, if at least some of the parameters and phenomena have a stochastic character.

The final function of a reservoir follows from the relationship between the time behaviour of water inflow and outflow. The release is determined by the water need which may either be planned or may show a random course. The natural inflow into the reservoir includes some random elements, such as the average annual flow (runoff from the watershed), the synchronous runoff from the watershed for certain periods in the respective years (months, etc.), the maximum or minimum annual flows, the beginning of the higher spring flow, the duration of the ice-cover on the river or reservoir, etc. All these elements can appear in any year to a certain extent.

Other elements of the natural runoff from the watershed have an essentially cyclic character, with only some random elements. This concerns mainly the annual course of the flow, with the characteristic cycle repeated every year (again within certain limits and with varying regularity according to the geographical region).

It is therefore obvious that the methods of mathematical statistics and probability theory can be applied more freely to elements with an annual interval, or for flood waves, than for changes in the flow and other hydrological parameters within the year where the sequence of the members and their arbitrary values cannot be freely interchanged.

Long-term fluctuations in hydrological phenomena were first of all considered to be completely random, i.e., without any statistical relationships between the individual years. It was only in 1936 that Yefimovich published a thesis that the correlation between runoff of neighbouring years should be taken into consideration and in 1959 Kritsky and Menkel proposed a method, and Gugli constructed nomograms, for the mathematical solution of over-year flow control including these correlations. The designs of impounding reservoirs indicated that even with flow fluctuations lasting many years a trend towards grouping of years with high- or low-flow periods could be observed.

The genetic concept of the hydrological phenomenon leads to the study of the reason for its cyclic nature based on the reasons for the cyclic character of its genetic elements (i.e., runoff in terms of precipitation, air temperature, etc.). Empirically we seek a coincidence between hydrological phenomena and a regularly repeated solar activity pattern. Such studies are not yet satisfactorily completed, but even the present results are encouraging (see Section 3.2). The runoff from a watershed is obviously influenced by so far unknown factors of a random nature, which are the reason why at present its long-term variability is considered to be more or less pseudocyclic.

For their mathematical expression of the relationships appearing as a result of random influences on the runoff and influences of inertia in the state of the watershed Kritsky and Menkel (1946) used processes (series, chains) developed by Markov (1951), which represent the generalization of a set of independent observations. The authors were of the opinion that the application of the simple Markov series would agree sufficiently well with the present knowledge about the correlation between the fluctuations in the solar activity pattern and those of the runoff.

2.2.1 *Markov processes*

Let us consider a series of observations. In each of them there might occur one, and only one, of k non-interchangeable cases $A_1^{(s)}, A_2^{(s)}, \ldots, A_k^{(s)}$ (the superscript marking the observation number). The series of observations creates a *simple Markov series*, if the probability of the appearance of case $A_i^{(s+1)}$ $(i = 1, 2, \ldots, k)$ in the $(s + 1)$th observation $(s = 1, 2, \ldots)$ depends only on which case occurred in the sth observation and is not changed by the knowledge as to which data on cases occurred in the earlier observations or which appeared later on. We could also speak about the physical system S which at each moment can be in one of the states $A_1, A_2, \ldots, A_k$ and changes its state only in the intervals $t_1, t_2, \ldots, t_n, \ldots$, while the probability of transition into an arbitrary state A_i $(i = 1, 2, \ldots, k)$ at the moment τ $(t_s < \tau < t_{s+1})$ depends only on the state the system was in at the moment t $(t_{s-1} < t < t_s)$ and does not change by reason of the fact that its states at earlier moments are known.

The Markov process, therefore, is a special case of a random process. If discrete time is used (as is usual in water-management reservoir design), it is called a *Markov chain*. By only taking into consideration the relationships between two neighbouring states, we arrive at the *Markov process of the first-order*, also called the *simple Markov process*. If we consider n relationships in the process we obtain the Markov process of the nth order or the *composite Markov process* (or *chain* in the case of discrete time).

The Markov series permit a quantitative comparison of the studied relationships and may be applied to water management problems. Most frequently we use only the simple Markov series, presuming a correlation between the neighbouring members of a series.

The transition from the series of independent random quantities

$$x_1, x_2, \ldots, x_i, \ldots$$

to the Markov standardized series (with the same indices)

$$y_1, y_2, \ldots, y_i, \ldots$$

follows from the presumption that the distribution of the probability of an arbitrary member in the ith place in the series is correlated to the value of the previous $(i - 1)$th member. If the correlation coefficient is r, the mean value of the ith member of the Markov series is

$$\bar{y}_i = r y_{i-1} \tag{2.19}$$

and its standard deviation

$$\sigma_{y_i} = \sqrt{1 - r^2} \tag{2.20}$$

In the case of a normal distribution $(\bar{x}_i = 0, \sigma_{x_i} = 1)$ we have

$$\frac{y_i - \bar{y}_i}{\sigma_{y_i}} = \frac{x_i - \bar{x}_i}{\sigma_{x_i}} = x_i$$

and therefore

$$y_i = \bar{y}_i + x_i \sigma_{y_i} = r y_{i-1} + x_i \sqrt{1 - r^2} \tag{2.21}$$

According to the given series of random quantities, we can calculate from equation (2.21) the individual members in the simple Markov series connected by the correlation coefficient of the neighbouring members r.

Let us consider the series of observed hydrological parameters as a simple Markov series with the correlation coefficient of the neighbouring members $r = r_1$, the correlation coefficient of members with distance two is $r_2 = r_1^2$ and that of members with distance three is $r_3 = r_1^3$, etc.

This *correlation (auto-correlation) function* therefore has the power form

$$r = r_1^\tau \tag{2.22}$$

where τ is the distance (time lag) in the sequence of correlated elements. It has a rapidly decreasing course, but with a positive r_1 it cannot pass to negative values (Fig. 2.14).

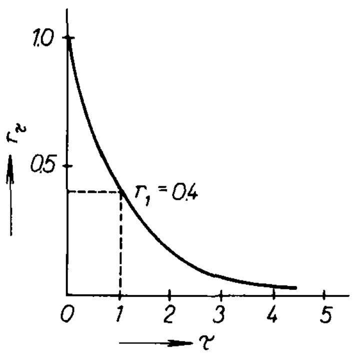

Fig. 2.14 Correlation power function $r_\tau = r_1^\tau$

We can then determine the parameters of the series by means of the expressions mentioned later (derivation: Kritsky and Menkel, 1946).

Standard deviation of the series

We investigated a series consisting of a large number (N) of samples from a Markov process, the size of each being equal to n.

$$\sigma = \frac{S^2}{1 - \dfrac{2r}{n(n-1)(1-r)}\left(n - \dfrac{1-r^n}{1-r}\right)} \tag{2.23}$$

in which S^2 is the mean value of the sample variances of all N samples.

Mean square error of the sample mean σ_n

A section chosen at random from a simple Markov series including n members is examined

$$\sigma_n = \frac{\sigma}{\sqrt{n}}\sqrt{1 + \frac{2}{n}\frac{r}{1-r}\left(n - \frac{1-r^n}{1-r}\right)} \tag{2.24}$$

where for σ we can substitute the expression from equation (2.23).

Correlation coefficient r between neighbouring members

We investigate the relationship between the sample correlation coefficient r_b and the population value of the correlation coefficient r.

$$r_b^2 = \frac{r^2 + \dfrac{2r}{n(n-1)(1-r)}\left(n - \dfrac{1-r^n}{1-r}\right)}{1 + \dfrac{2}{n(n-1)(1-r)}\left(n - \dfrac{1-r^n}{1-r}\right)} \tag{2.25}$$

Arbitrarily chosen sections can be studied as a part of the simple Markov series, by calculating first, on the basis of the equation (2.25), the correlation coefficient r and then determining the standard deviation σ according to equation (2.23) and the mean square error of the average sample mean σ_n according to equation (2.24).

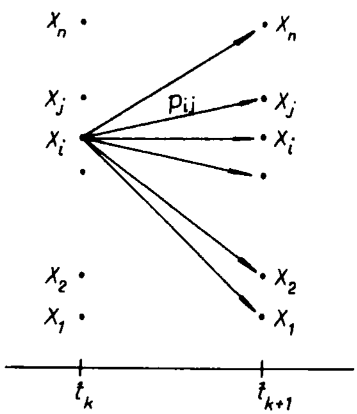

Fig. 2.15 Scheme of transition probability

The Markov processes are also studied with the aid of a *transition probability* which is the conditional probability p_{ij} that from a certain state X_i the process passes at the following moment into another state X_j. If at any moment the process has a finite number of n states $X_1, X_2, \ldots, X_n$, then n is the probability p_{ij} that the state X_i $(i = 1, 2, \ldots, n)$ passes to states $X_1, X_2, \ldots, X_n$, or $j = 1, 2, \ldots, n$. The number of transition probabilities from each state at the moment t_k into each state at the moment t_{k+1} is n^2 (Fig. 2.15).

The transition probabilities are recorded clearly in the transition matrix which naturally in this case is of the square type $(n \cdot n)$:

$$\boldsymbol{p}_{i,j} = \begin{bmatrix} p_{11}, & p_{12}, & \ldots, & p_{1n} \\ p_{21}, & p_{22}, & \ldots, & p_{2n} \\ \vdots & & & \\ p_{n1}, & p_{n2}, & \ldots, & p_{nn} \end{bmatrix} \tag{2.26}$$

The sum of the values in a row of the matrix is $\sum_{j=1}^{n} p_{ij} = 1$, since it expresses the probability of the transition from state X_i to all n states.

Each column of the matrix shows the probability of the transition from all n states into the state X_j; for this reason, in general their sums vary.

2.2.2 *Application of probability and statistical methods in water management*

The basic hydrological material for the solution of water-management problems in reservoirs is the result of measurements performed *in situ*; but even long hydrological series do not usually include extreme values, which is why the observed hydrological material is corrected by the probability theory and mathematical statistics. Their first application in water-management calculations is in the processing of basic data (see Chapter 3). Elements with one or more common symbols are collected into populations and various characteristic values are determined for them, such as arithmetic mean $\bar{x}$, extreme values x_{max} and x_{min}, variation ranges $x_{max} - x_{min}$, the mean and the standard deviation, the frequency of occurrence, the variation coefficient C_v, the skewness coefficient C_s, the theoretical exceedance probabilities curve, the correlation between two or more statistical sets, etc.

The difference between the empirical measured values and their statistical processing is quite obvious if we compare the real and the theoretical exceedance probability curves for the flows. The theoretical curves not only give a more reliable guarantee, especially for extreme flows, but the statistical methods also permit a generalization of the results and their application to other hydrological conditions on other rivers. Thus we have a tool with which to solve the water-management problems of rivers with only few measured data.

Statistical populations include members with one or more common symbols; but their values are random. Such an indicator of the annual runoff is, e.g., the total annual runoff (without any relationship to the runoff in the neighbouring years), the maximum annual flow, floods of various magnitude, etc. Mathematical statistics so far used in hydrology have mostly been applied to annual runoff and flood waves.

In order to be able to solve questions of irrigation and drainage, hydrological parameters must be found (precipitations, runoff, temperatures, etc.) in the vegetation period. For hydro-power problems we need to know the flow in the winter months, for pollution prevention of the rivers we need to know the flow at the time of the maximum load of waste water, etc.

These seasonal discharges of the respective years also form statistical populations. With these we can work in the same way as with annual flow. The common indicator for all set members (i.e., of the runoff, precipitation, etc.) is the value at a certain period of the year.

The second field in which probability and statistical methods are used in water management, is for calculation of the flow control of reservoirs. The rules for inflow and withdrawal from the reservoir are generally independent of one an other; only in some cases can a certain relationship exist between them. Otherwise, withdrawals are determined by the need for water, while inflow into the reservoir is mainly determined by natural laws and they can be processed by statistical methods.

As the character of random quantities is especially evident in annual hydrological parameters, statistical methods were used for the first time to determine the over-year components in reservoir volume. The statistical set was used to compile yearly flows (Hazen, 1914). It is logical that the main factor for the determination of reservoir volumes on the basis of statistically processed hydrological data is the over-year flow control.

An annual flow control each year furnishes a complete closed reservoir cycle – filling and withdrawal. This means that for each of the n years in a chronological flow series we have to calculate the dependable withdrawal O_p for a given storage volume V_z, or vice-versa, to get n results. If the series has several dozen members, these create the same number of examined and mutually independent dependable withdrawals (or necessary storage volumes), a large enough set to be processed and to supply results which can be generalized.

If the outflow is controlled over several years, the number of groups of years decisive for the evaluation of the results is much lower than the total number of years and therefore the number of independent results is also much smaller. To deduce general conclusions from these results is therefore also more difficult; even more so, if the coefficient of the safe yield $\alpha = O_p/Q_a$ approaches unity or the longer the period of the flow control is. It is obvious that a prolongation of the control cycle means a reduction in the reliability of the hydrological series regarding their ability to reflect the regime to be expected in the future operation of the reservoir. Therefore, it becomes necessary to compile a synthetically larger number of flow combinations than that offered by the observed hydrological series and to solve with them the problems arising from over-year cycles. The tools for this are supplied by probability theory and by mathematical statistics.

The reservoirs completed during the last twenty years have often volumes for a flow control over several years. Probability methods were necessary in their design.

Since one chronological series is more specific than the character of the parameters derived from the same number of values, regardless of their chronological sequence, the method based on the probability distribution of the basic hydrological data may offer a more reliable result than the method using one real chronological arrangement of these data.

Judging the results of the solution from the point of view of the extent to which the water supply is guaranteed, it remains impossible to say in advance (if there is no analogy with other longer series) that the shorter period has a lower dependable yield (in %) than the longer one. If the observations happen to be performed in unfavourable years, an identical dependable yield would mean lower withdrawals than those based on a longer series which would include years with more favourable flows than in the unfavourable period considered above. If, however, the observations happen to take place in a period with a favourable flow, the same dependable yield results in withdrawals that are higher than in the longer series.

Short hydrological series might therefore furnish results that deviate considerably from the results of the longer series. The deviation may be in either direction. This is the reason why we have to evaluate the representative character of the series; for instance, its analogy with another longer series.

The results gained from a chronological series will be more susceptible to the incorrect selection of hydrological data than results obtained by probability methods. Naturally, not even probability methods can furnish correct results, if the data are incorrect or unverified.

The relationships between direct methods in observed chronological hydrological series and indirect methods based on general hydrological statistical characteristics, can be defined in the following way:

1. On the basis of the present knowledge of the respective methods, it can be recommended that mainly indirect methods should be used. Direct methods can only be applied for solving the function of reservoirs with a low coefficient of safe yield and a lower reliability required, e.g., mainly with a short-term or annual flow control, for the following reasons:

(a) For the usual length of hydrological series (30 to 40 years) control with a short cycle furnishes such a large number of mutually independent results that (after probability evaluation) the reliability of the resulting parameters can be presumed.

(b) For an annual or low over-year flow control, the entire storage volume of the reservoir, or its decisive part, is created by the seasonal part of the reservoir volume resulting from the annual variability cyclic flow.

2. Solutions in chronological series are clear and easy to survey, which is of advantage for the explanation of the release control by reservoirs and for the solution of complicated water-management problems. However, the reliability of such a solution always has to be assessed.

3. The flow characteristics as measured by hydrometry without processing should not always be accepted as a basis for water-management solutions. Time series characteristics should be assessed more accurately for values in a hydrological series which are not sufficiently realiable – especially the extreme values. For short available periods, especially, they should be processed in order to eliminate random irregularities; such assessments use statistical methods. The measured time series characteristics make it possible to use genetic methods of deterministic hydrological research for analyses of the outflow conditions (important, e.g., for outflow forecasts).

4. The question of safe yields in flow control by reservoirs cannot be solved without probability methods.

5. When using general statistical characteristics, the entire storage volume is

divided into the over-year component and the seasonal component; the seasonal component is determined by dividing up the outflow during the year.

6. General solutions based on probability methods using statistical characteristics have the advantage that the relationship between the parameter values of the reservoir and its regime can also be applied to other rivers. They are therefore useful in regions where there is a lack of hydrological data.

7. General methods make it simpler and easier to solve flow regime problems, as the calculation of statistical characteristics is easier than finding solutions by simulation.

8. Synthetic hydrological series combine the advantages of the balancing method in chronological series (simulation) and of the probability theory (see Chapter 3).

9. Probability methods are indispensable for the solution of problems of water-management systems with reservoirs.