5 OVER-YEAR RELEASE CONTROL

Over-year reservoirs are currently used to solve the conflict between increasing water demand and limited resources. They form one of the most important links in systems which supply various users with water from surface resources. Growing demands and limited resources must be balanced by the regulation of natural discharges, which can only be done with the help of over-year discharge control.

Let us consider a reservoir with a storage capacity (in this chapter storage capacity means active storage capacity) which has to ensure a relatively high yield. The coefficient of the relative yield α ($= O_p/Q_a$) is e.g. equal to three quarters, the required reliability $P_0 = 99\%$. In the hydrological conditions pertaining in Czechoslovakia, low-flow periods occur during which, from time of the beginning up to the stage of

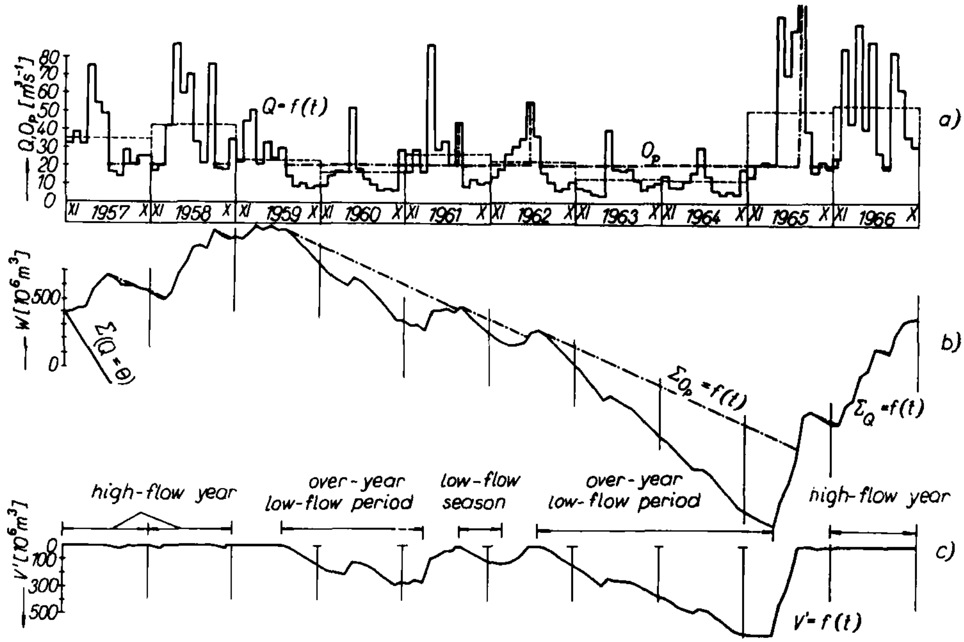

Fig. 5.1 Stages of the work regime of a reservoir with an over-year release control
(*a*) time pattern of inflow and release; (*b*) illustration of the reservoir function by means of discharge and release mass curves; (*c*) time pattern of the emptying of the storage capacity

complete emptying of the storage capacity takes longer than one year. The subsequent filling of the latter can take several years. The greatest emptying of the storage capacity of a reservoir does not take place at the end of the low-flow season of the first year of a longer period of low flows, but in one of the following years; the excess discharge of the high-flow seasons in between is not sufficient to refill the storage capacity (Fig. 5.1). Low-flow periods lasting for several years include an annual cycle which must be taken into account in the design. At the same time, the discharge is balanced for a short period, e.g., a month, a week, a day.

In high-flow years, the storage capacity of a reservoir is emptied to some degree during low-flow periods of the year, but it is refilled in the following high-flow period. In such years only a seasonal or annual discharge regulation typical for within-year (seasonal) release control is applied.

In high-flow years, the low-flow periods last only a few days and the inflow to a reservoir does not drop below the value of the outflow (withdrawal). The storage function of the reservoir is then made use of only in the few days of a low-flow period so that the storage capacity is mostly full. If several high-flow years follow one another, the storage capacity of a reservoir is only used to a limited extent, even in over-year release control.

The storage capacity of a reservoir is designed on the basis of data obtained from a sequence of several low-flow years. Therefore the design need be based on the long-term flow-regime characteristics.

An over-year reservoir cycle can occur for reservoirs with diversion (withdrawal), with river flow regulation, with reservoir in a cascade, and for those working in a system. This chapter deals with reservoirs with direct withdrawal.

The storage capacity with an over-year release must be designed in such a way as to accumulate water in high-flow years and to raise the release (withdrawal) in low-flow years (when $Q_r < O_p$).

The theoretical exceedance curves indicate that most of the cases where a reservoir has a storage function with direct withdrawal can theoretically be included in the over-year outflow control. Where the probability of exceedance P (abscissa) is very close to the value 100%, the ordinates of the theoretical curve of exceedance (mean

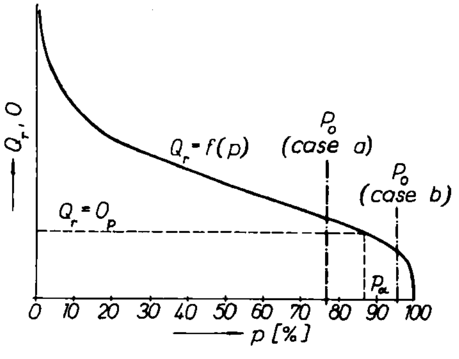

Fig. 5.2 Considerations about release control in the exceedance curve of mean annual discharges (inflows to a reservoir)

annual discharges) are close to zero (presuming the validity of the Pearson type III distribution where $C_s = 2C_v$ corresponds to the value $P = 100\%$ zero value of the mean annual discharge). Theoretically there is always a non-zero interval $(P_\alpha; 100\%\rangle$, Fig. 5.2. In this interval the values Q_r of the mean annual discharges are smaller than the required mean annual withdrawal from a reservoir O_p [for P_α it holds that $Q_r(P_\alpha) = O_p$]; it is therefore necessary to have a certain volume from the high-flow years in the storage capacity of a reservoir, which means that the design working cycle of water supply is an over-year cycle. If discharge regulation is not extensive, the probability that the mean annual discharge will be smaller than the withdrawal is very small; the range of the interval $(P_\alpha; 100\%\rangle$ is, in this case, close to zero. To disregard this influence does not mean that it will cause an unacceptable error and we are therefore justified in using the method which is typical for within-year release control.

With the design reliability P_0, the following cases can occur (Fig. 5.2):

(a) $P_\alpha \geqq P_0$, which might be considered to be the sign of a within-year (seasonal) release control. To ensure the required withdrawal with the given reliability, it would suffice to stabilize the uneven discharge distribution within all the years of the interval $\langle 0; P_0\rangle$ in such a way as not to cause a failure in the water supply. It can, however, be presumed that in some years which, by the value of their mean annual discharge are in the interval $(P_0; P_\alpha\rangle$, withdrawal from the reservoir without any failure would be required; in an over-year relase control $(P_\alpha; 100\%\rangle$ over-year low-flow periods can occur in which the reservoir volume designed for discharge regulation in very low-flow seasons within the framework of within-year release control, will suffice to stabilize the discharge.

It is therefore correct to consider this case, too, as an over-year release control; however for $P_0 \rightarrow 100\%$ the method of within-year release control can also be used.

(b) $P_\alpha < P_0$, which is sufficient condition for over-year release control. In practice the design reliability is usually very high ($P_0 = 95 \div 99\%$, rarely less). Then even if $P_\alpha < P_0$ the interval $(P_\alpha; 100\%\rangle$ has so small a range that the influence of over-year release control on the characteristics of the storage function cannot be expressed quantitatively and a simpler process must be used.

Division into within-year (seasonal) and over-year outflow control is of great practical importance. In over-year outflow control the design size of the storage capacity is given by low-flow periods lasting several years. These low-flow years can vary as to their number, the size of the mean discharge and the variability of flow during the year.

The longer the reservoir cycle (i.e., the more extensive the regulation of the discharge), the more thoroughly the changes in long-term characteristics of inflow to a reservoir will have to be studied. At the present state of knowledge, the most expedient method is the study of synthetic sequences of mean annual discharges from which a mathematical model can be elaborated. In hydrological conditions with irregular regimes of high-flow and low-flow periods attention must be paid to seasonal changes in the inflow.

In within-year release control we study the variability of discharges within the respective years; the relationship between high-flow and low-flow years in their time sequence need not be studied. The complexities of changes in discharge during the course of a year, make it impossible to create a typical design annual hydrograph; here too probability methods should be used to determine the characteristics of release control.

Hydrological data should therefore be prepared and other factors considered (e.g., water losses from the reservoir), according to the type of release control (see Chap. 3.4).

Methods used for over-year release control can be divided into two groups:

(a) methods based on the separation of the over-year and seasonal changes in the discharges, i.e., on the division of the over-year and within-year components of a reservoir storage capacity,

(b) methods based on the characteristics of release control, including the laws of the discharge regimes as a whole; for this, synthetic discharge time series are used which reflect, besides the long-term variability, also the variable discharges in the respective years.

To resolve the consequences of over-year and seasonal discharge fluctuations separately, two methods can be used:

- analytical methods, which determine the influence of over-year discharge fluctuations on the release control parameters, on the basis of the general statistical characteristics of mean annual discharges (Q_a, C_v, C_s) and the correlation function;
- methods using synthetic pseudo-chronological series of mean annual discharges.

5.1 METHOD BASED ON ANALYTICAL SOLUTIONS OF THE STORAGE FUNCTION

As in complicated hydrological conditions only the influence of long-term discharge changes can be determined analytically, it is necessary to divide the storage capacity of a reservoir V_z into two parts, i.e. the *over-year component* V_z^v and the *within-year (seasonal) component* V_z^s so that

$$V_z = V_z^v + V_z^s \tag{5.1}$$

The *over-year component* of the storage capacity is determined from the mean annual inflows to a reservoir and the withdrawals, the *seasonal component* should express the demands on the storage derived from the variability of inflow and withdrawal during the year. In expressing mean annual discharges in water years, the seasonal component in low-flow periods lasting several years is usually divided into two parts (Fig. 5.3).

Figure 5.3 illustrates a case in which the two parts of the seasonal component are given by the low-flow seasons of the years which either precede (year 1) or follow (year 4) a low-flow period of several years. The size of the seasonal component can also be influenced by fluctuations of discharges in the years within the low-flow period.

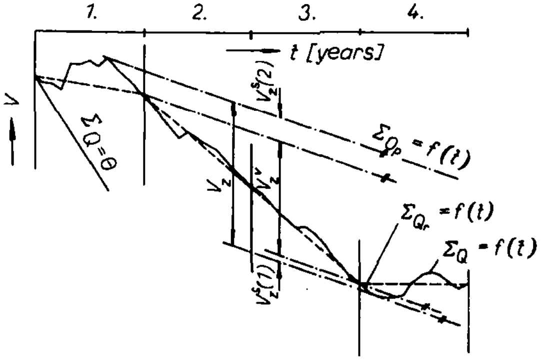

Fig. 5.3 Over-year and seasonal component of the active storage capacity

Equation (5.1) or equation (5.1′) in relative values of storage capacity

$$\beta_z = \beta_z^v + \beta_z^s \tag{5.1'}$$

theoretically has an infinite number of solutions, as various combinations of the sums $V_z^v(O_p, p_1) + V_z^s(O_p, p_2)$, or $\beta_z^v(\alpha, p_1) + \beta_z^s(\alpha, p_2)$ can be applied to the unique solution of the function $V_z = f(O_p, p)$ or $\beta_z = f(\alpha, p)$. Generally $p \neq p_1 \neq p_2$. Therefore, another condition has be introduced for the unique solution of the over-year and seasonal component. Usually the reliability of the over-year component is chosen as equal to the design reliability of water supply P; to this is added the seasonal component so as to meet the condition

$$V_z(O_p, P) = V_z^v(O_p, P) + V_z^s(O_p, p_2) \tag{5.2}$$

regardless of the reliability size p_2 of the seasonal component itself.

This definition of the over-year and seasonal component of the storage capacity for over-year release control makes it possible to solve separately the relationships between the characteristics of release control O_p, V_z^v, P or α, β_z^v, P, resulting from the variability of the mean annual inflows to a reservoir.

5.1.1 Over-year component of the storage capacity

The law of the sequences of mean annual discharges, presuming a *stationary hydrological regime*, is defined by basic statistical characteristics:

- long-term mean discharge Q_a,
- variation coefficient of mean annual discharges C_v,
- skewness coefficient of mean annual discharges C_s,
- correlation function $r(\tau)$.

The first three characteristics are sufficient to define the theoretical curves of exceedance; the correlation function gives the *internal sequence structure.*

If we neglect the influence of the correlation function (Kritsky and Menkel, 1932), in determining the over-year component, it is sufficient to know the duration of the design low-flow period n (number of years) and the mean discharge in this period $Q_n(p)$ (inflow into a reservoir), with the probability of exceeding p being equal to the design reliability P, according to the number of years without failure. Then

$$V_z^v = nT_r[O_p - Q_n(p)] \qquad [\mathrm{m}^3] \tag{5.3}$$

or in relative values

$$\beta_z^v = n[\alpha - k_n(O_p)] \tag{5.3'}$$

where $k_n(p) = Q_n(p)/Q_a$ and T_r is the number of seconds per year ($T_r = 31.6 \cdot 10^6$ [s]).

As the number of years of the design low-flow period is not known in advance and as it changes with O_p (or α) and with p, a trial-and-error method has to be applied.

Let us presume that the design low-flow period is only a *one-year period* ($n = 1$). Then it is sufficient to read the value $Q_1(p)$ for $p = P$ from the theoretical curve of exceeding mean annual discharges $Q_r = f(p)$ and to calculate

$$V_z^v(1) = T_r[O_p - Q_1(p)] \tag{5.4}$$

For a *two-year period* we calculate first, from the sequence of mean annual discharges, $Q_{r,1}, Q_{r,2}, Q_{r,3}, \ldots, Q_{r,n}$ the mean values for two years:

$$Q_1(2) = \frac{Q_{r,1} + Q_{r,2}}{2},\ Q_2(2) = \frac{Q_{r,2} + Q_{r,3}}{2}, \ldots, Q_{n-1}(2) = \frac{Q_{r,n-1} + Q_{r,n}}{2} \tag{5.5}$$

From the values $Q_i(2)$ we compile an empirical exceedance curve to which we fit a theoretical curve and for $p = P$ we read the mean discharge in the design two-year period $Q_2(p)$. Then

$$V_z^v(2) = 2T_r[O_p - Q_2(p)] \tag{5.6}$$

The procedure is the same for three-year, four-year, etc., periods.

For *k-year design low-flow period*

$$V_z^v(k) = kT_r[O_p - Q_k(p)] \tag{5.7}$$

From a certain value k the values $V_z^v(k)$ start to decrease, however, if $O_p < Q_a$ ($\alpha < 1.0$). When $O_p \geqq Q_a$, then the right-hand side of equation (5.7) exceeds all limits with the increase of k.

The design size of the over-year component of the storage capacity will then be

$$V_z^v = \max\{kT_r[O_p - Q_k(p)]\} \tag{5.8}$$

If the terms of the synthetic sequence of mean annual discharges [correlation function $r(\tau) = 0$ for $\tau > 0$; $r(\tau = 0) = 1$] are mutually independent, it holds that

$$C_v(k) = \frac{C_v}{\sqrt{k}}; \qquad C_s(k) = \frac{C_s}{\sqrt{k}} \tag{5.9}$$

If $r(\tau) \neq 0$ (also for $\tau > 0$) the variation coefficient $C_v(k)$ must be determined from the relationship

$$C_v(k) = \frac{C_v}{\sqrt{k}} \sqrt{1 + 2\left(r_1 \frac{k-1}{k} + r_2 \frac{k-2}{k} + r_3 \frac{k-3}{k} + \ldots\right)} \tag{5.10}$$

and

$$C_s(k) : C_v(k) = C_s : C_v$$

If we neglect r_2, r_3, etc., then

$$C_v(k) \doteq \frac{C_v}{\sqrt{k}} \sqrt{1 + 2r_1 \frac{k-1}{k}} \tag{5.10'}$$

Figure 5.4 illustrates the results of the calculations of the over-year component of the storage capacity for various relative yields α with a reliability of $P = 97\%$, $C_v = 0.427$, $C_s = 0.8$, presuming that equation (5.9) is valid. The dashed lines show the relationship of $\beta_z^v = f(k)$ for $\alpha = 0.8$, determined directly from the curves of exceeding mean discharges in the respective k-years. The results differ greatly ($\beta_z^v = 0.86$ as compared to the value 0.66). Simple and logical principles were used to prove that the curves of exceeding mean annual discharges are suitable to determine the over-year component of the storage capacity. As the problem can be solved

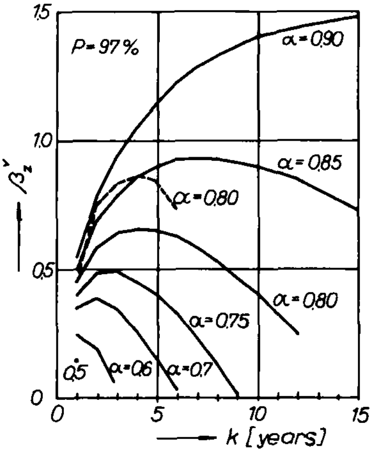

Fig. 5.4 Calculations of the over-year component of the storage volume by means of the exceedance curves of mean discharges during n years (Kritsky and Menkel, 1930)

regardless of the observed chronological discharge series, we obtain much more general results, which include any discharge conditions. The real series $Q_r = f(t)$ only helps to determine the statistical characteristics and the correlation functions, which are considered to be the estimate of the parameters of the basic population. As the changes in the values of the statistical characteristics (especially C_v) greatly influence the results of the calculations of the over-year release control (see Section 5.3), all possible means (Chap. 3) must be used to determine the most representative values Q_a, C_v, C_s and $r(\tau)$.

Mean discharges with a given probability of exceedance in the respective k-years, which are the basis of the calculations, make it possible to get to know more comprehensively the laws of the river regime and can also be used for other purposes, such as a "safe" forecast of the first filling of a reservoir, for a statistical evaluation of low-flow periods lasting several years during operations, etc. One of the advantages of this method is that it can determine the relationship $\beta_z^v = f(k)$, which helps to evaluate some of the other influences on the size of a reservoir in relation to the duration of a reservoir refill cycle, e.g., water losses, etc.

Other methods of assessment of the over-year component of the reservoir storage capacity are based on general statistical characteristics of the law of inflow to a reservoir and use the theory of stochastic processes.

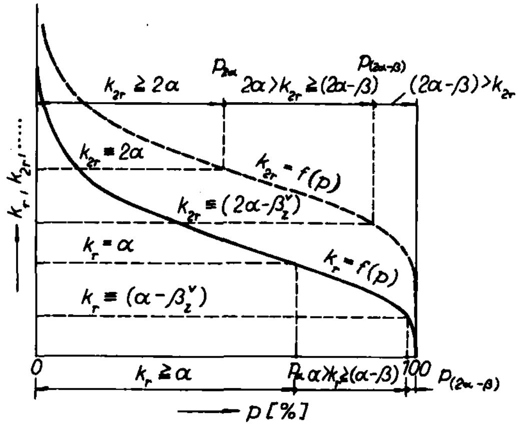

Fig. 5.5 Calculations of the over-year component of the storage volume by means of the composition of exceedance curves of mean discharges during n-years (Kritsky and Menkel, 1935)

The principle of Kritsky's and Menkel's "second" method (derived in 1935 and generalized in 1956 to 1957) is based on the separation of the intervals of the years with failures and without failures in the curve of exceeding mean annual discharges $Q_r = f(p)$, or annual modulus coefficients $k_r = f(p)$ and on a detailed analysis of interval in which a failure to supply water can or need not occur (interval of conditional failure years).

Stated is the curve of exceeding the annual modulus coefficients $k_r = f(p)$ (Q_a, C_v, C_s and the type of a suitable theoretical function), the relative yield α (given O_p)

and the relative over-year component of the storage capacity β_z^v. The unknown quantity is, therefore, reliability P.

In the exceedance curve (Fig. 5.5) we can separate the years that must be without failure, in which $k_r \geqq \alpha$ in the interval $\langle 0; p_\alpha \rangle$, and the years in which failures occur in which $k_r < (\alpha - \beta_z^v)$ in the interval $(p_{(\alpha-\beta)}; 100\%\rangle$. In the remaining interval $(p_\alpha; p_{(\alpha-\beta)}\rangle$ there are the years for which $\alpha > k_r \geqq (\alpha - \beta)$ which can be without failure, if the water in a reservoir suplements the deficit $(\alpha - k_r)$, or with failures, if the storage volume is so low that it is unable to cover the deficit. To determine the difference between the years with failures and those without failures in this interval, a two-year composition of the years in the interval $(p_\alpha; p_{(\alpha-\beta)}\rangle$ and from the previous years must be calculated, then a three-year, four-year, etc., period until all years are divided into years with and without failures to supply water, whereby we obtain the reliability P sought.

Presuming that the sequence of the mean annual discharges is independent $[r(\tau) = 0$ for $\tau > 0]$, we obtain the exceedance curve of two-year modulus coefficients $k_{2_r} = f(p)$ by combining the exceedance curves $k_r = f(p)$ for $p = 0$ to 100% (curve I) and $k_r = f(p)$ for p in the interval $(p_\alpha; p_{(\alpha-\beta)}\rangle$ (curve II), which must be extended to interval (0; 100%) in such a way that value p_α is ascribed $p = 0$ and value $p_{(\alpha-\beta)}$ the limit $p = 100\%$; between those boundary values the scale p is linear. Curves of exceedance (or reliability) can be compiled by various, mostly graphical or graphical-numerical methods (Votruba and Broža, 1974).

In exceedance curve $k_{2_r} = f(p)$ we again determine three intervals of probability of exceeding (Fig. 5.5):

- interval $\langle 0; p_{2\alpha} \rangle$ of two-year periods without failure,
- interval $(p_{(2\alpha-\beta)}; 100\%\rangle$ of a two-year period necessarily with failures,
- interval $(p_{2\alpha}; p_{(2\alpha-\beta)}\rangle$ of two-year periods with conditional failures.

As the exceedance curve $k_{2_r} = f(p)$ helps to decide in which years failures will or will not occur in interval $(p_\alpha; p_{(\alpha-\beta)}\rangle$, the width of the respective intervals of the exceedance probability curve $k_{2_r} = f(p)$ must be multiplied by the width of interval $(p_\alpha; p_{(\alpha-\beta)}\rangle$, i.e., the value $0.01 \cdot (p_{(\alpha-\beta)} - p_\alpha)$, if p is given in percent.

Then in the interval of years of inevitable failure $(p_{(\alpha-\beta)}; 100\%)$ a supplement will be added, resulting from the years of inevitable failure, equal to the value of

$$S_2 = (100 - p_{(2\alpha-\beta)}) \cdot 0.01(p_{(\alpha-\beta)} - p_\alpha)$$

and in the interval of a conditional failure two-year periods of the width

$$(p_{(2\alpha-\beta)} - p_{2\alpha}) \cdot 0.01(p_{(\alpha-\beta)} - p_\alpha)$$

another year must be added to compile a curve of exceeding a three-year modulus coefficient $k_{3_r} = f(p)$, in which we again decide which years will be years of inevitable failure and which will be without failure.

The increments of years of inevitable failure rapidly converge towards zero so that in practical applications the number of steps is small.

The sum of the width of intervals of inevitable failure years in the respective steps $\sum S_i$, gives us the unreliability of the water supply (in percent); by bringing this value up to one hundred we obtain the reliability of the required yield α (or O_p) with the given value β_z^v (or V_z^v).

If we apply the method described above for a series of values β_z^v with a given relative yield α, we can construct the relationship $\beta_z^v = f(p)$ and determine the resultant size $\beta_z^v(P)$ for the design reliability of water supply P (occurrence-based).

By repeated calculations for several values of α, data are obtained for the construction of the relationship $\alpha = f(p)$ with a constant β_z^v and $\beta_z^v = f(\alpha)$ with a given P.

Pleshkov (1939) constructed graphs for the *mutual independence of mean annual discharges* $[r(\tau) = 0$ for $\tau > 0$; $r(\tau) = 1$ for $\tau = 0]$ which express the relationship $\beta_z^v = f(C_v)$ for values of α (from 0.15 to 0.90 with a step 0.05) and p (75, 80, 85, 90, 95, 97%), presuming that $C_s = 2C_v$ (see, e.g., Votruba – Brožа, 1966, p. 198).

If the correlation function $r(\tau)$ *is not zero*, the probability of encountering an arbitrary value k_r (generally speaking, a k-year modulus coefficient) from the interval of conditional failure years (or conditional failure k-year periods) with an arbitrary value k_r on the curve of exceeding annual modulus coefficients (from 0 to 100%) is not the same. This must be taken into consideration when constructing exceedance curves for k-year modulus coefficients, which will be conditional.

Conditional parameters of curves of exceedance and of the conditional correlation coefficient for k-years must be estimated from unconditional parameters valid for $(k - 1)$-years, for $(k + 1)$-years from parameters for k-years, etc. Presuming the stationarity of the random sequence of mean annual discharges and the same type of distribution of conditional and unconditional curves. Kritsky and Menkel (1959) derived the necessary relations valid for the general correlation function.

In the conditional curves of exceeding the intervals without failures, necessarily with failures and conditionally with failures k-years are then determined; calculations then continue as for a zero correlation function.

Kritsky and Menkel (1959) used the following correlation function

$$r(\tau) = r_1^{\tau}\,, \quad \text{resp.} \quad r_k = r_1^k \tag{5.11}$$

For this function, with $r_1 = 0.30$ and $C_s = 2C_v$, Gugli (1959) constructed graphs $\beta_z^v = f(C_v)$ for a given α and p, which are formally identical with Pleshkov's graphs.

By analysing empirical correlation functions of mean annual discharges in the Labe catchment, Nacházel (1965) reached a relationship expressing a damped harmonious motion

$$r_\tau = \frac{5}{3} r_1\, e^{-0.1\tau} \cos\frac{2\pi}{15}\ (\tau + 1) \tag{5.12}$$

Figure 5.6 compares the course of the two correlation functions with $r_1 = 0.3$. The *power correlation function* always leads to positive values of r_τ, even though, roughly from value r_4, one can speak of a practical independence. It reflects the greater probability that low-flow and high-flow years appear in groups and puts greater demands on the over-year component of the storage capacity than in the case of independent mean annual discharges.

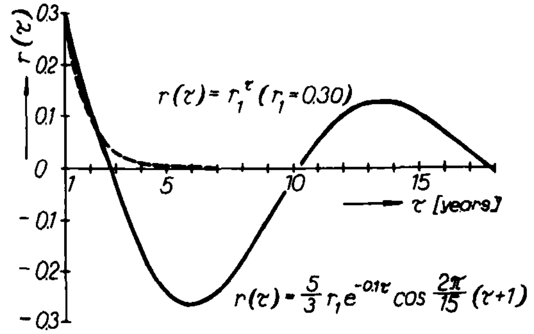

Fig. 5.6 Comparison of the behaviour of the correlation function expressed by the equation of the damped harmonious motion and the power correlation of the function

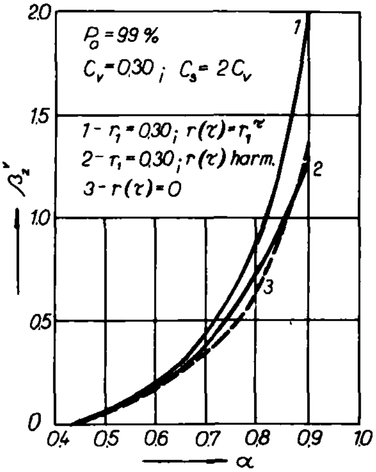

Fig. 5.7 Influence of the correlation function of mean annual discharges on the size of the over-year component of the storage capacity (relationship $\beta_z^v = f(\alpha)$)

The *correlation function in the shape of a damped harmonious motion* (Fig. 5.6) permits an *indirect correlation dependence* (with a negative extreme of r_6). This fact is reflected mainly in a clearly over-year cycle of several years, i.e., one that requires a high degree of regulation α, where the demands on the over-year component will be smaller than for the power correlation function. A comparison of the relationship $\beta_z^v = f(\alpha)$ throughout the whole course, introducing various correlation functions, can be found in Fig. 5.7 (Nacházel, 1965).

Several sets of diagrams help to determine the over-year component of the storage capacity β_z^v, with given values of α and P, based purely on statistical characteristics and the correlation functions of mean annual discharges (e.g., Svanidze, 1964; Reznikovski *et al.*, 1969; Nacházel, 1973).

For their close relationship to our hydrological conditions we chose the graphs constructed by K. Nacházel with the correlation function (5.12) for $r_1 = 0.30$ (Fig. 5.8); Pleshkov's and Gugli's graphs can be found in, e.g., Votruba and Broža (1966). These graphs make it possible to determine very easily one of the three quantities α, β_z^v, P, if the other two are given and have the known characteristics C_v, C_s and $r(\tau)$.

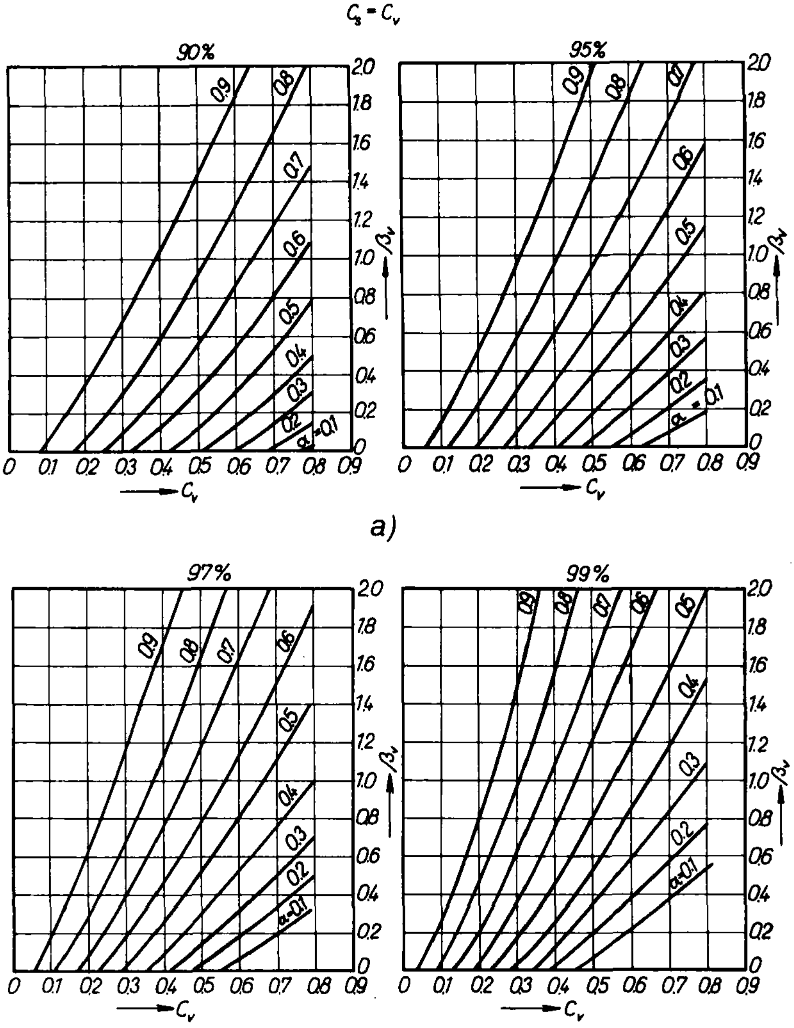

Fig. 5.8 Graphs to determine the over-year component of the storage capacity (Nacházel, 1973), $r(\tau)$ – damped harmonious ($r_1 = 0.30$)
(a) $C_s = C_v$; (b) $C_s = 2C_v$; (c) $C_s = 3C_v$

If the estimated quantities α, P and also C_v, C_s, r_1 are not identical with the quantities in the graphs, then the result can be obtained by interpolation (the best way is graphically after plotting several points of the studied relationship).

Savarenski (1935 – 1951), and P. A. P. Moran and others, chose a different method for release control.

This method is based on the probability with which a certain chosen water volume will be accumulated in the storage capacity of a reservoir at the turn of two years. If this probability is determined for every size of the filling of the storage capacity

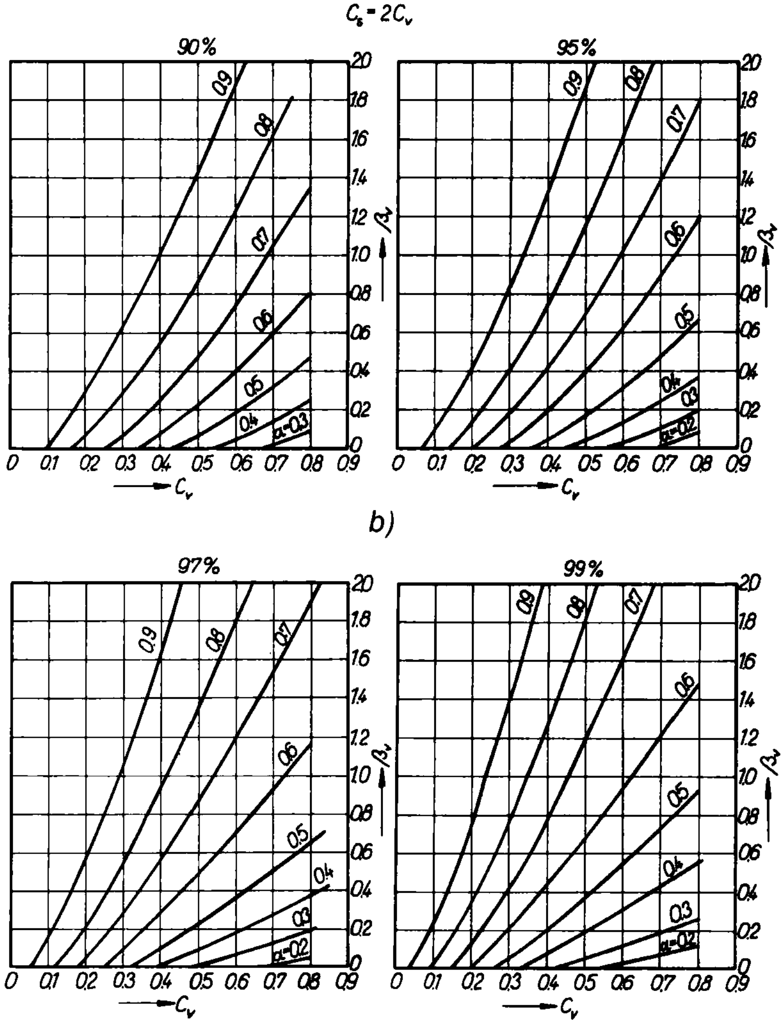

Fig. 5.8 Graphs to determine the over-year component of the storage capacity (Nacházel, 1973).
$r(\tau)$ – damped harmonious ($r_1 = 0.30$)
(a) $C_s = C_v$; (b) $C_s = 2C_v$; (c) $C_s = 3C_v$

(from zero to maximum value), the rule for the distribution of the water volume in a reservoir is obtained. Savarenski (1950), Moran (1959), Gould (1961), White (1963), Lloyd (1963) and others introduced a simplified pattern of the discharge within a year into the calculations, but due to the complex conditions of Middle European rivers this grossly distorts the results.

We therefore use *Moran's formulation for the over-year component of the storage capacity of a reservoir*, only slightly adjusted for practical applications.

Let us presume that the mean annual discharges create a random sequence which

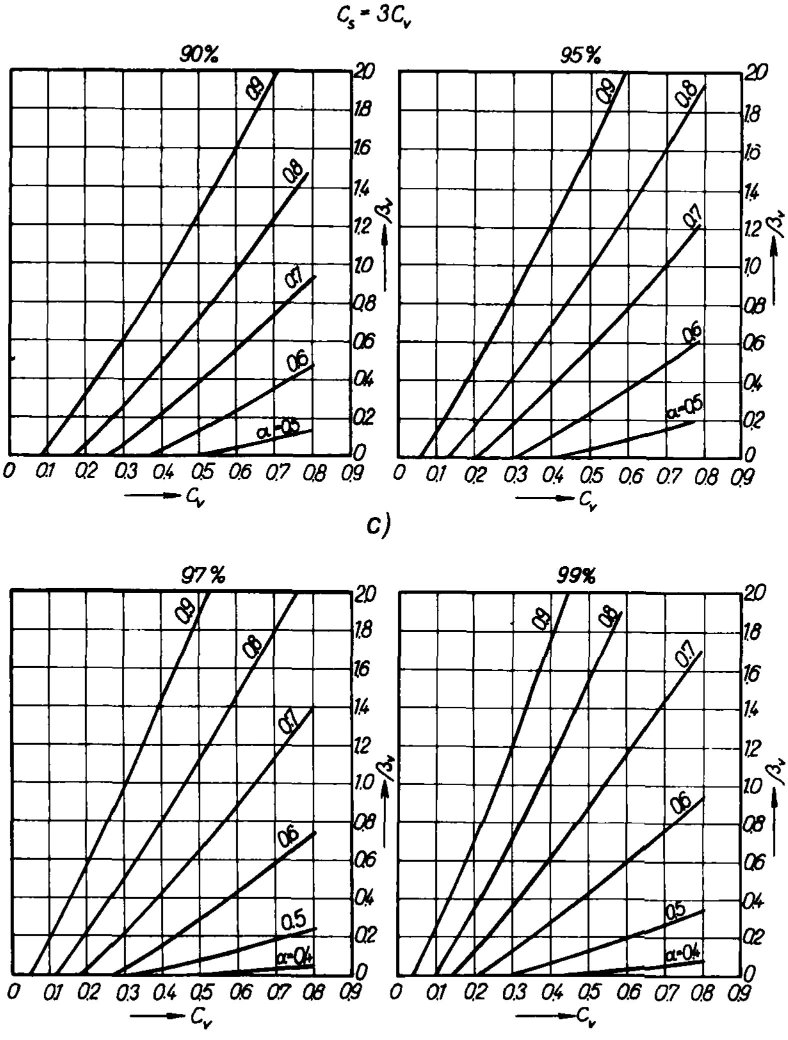

Fig. 5.8 Graphs to determine the over-year component of the storage capacity (Nacházel, 1973), $r(\tau)$ – damped harmonious ($r_1 = 0.30$)
(a) $C_s = C_v$; (b) $C_s = 2C_v$; (c) $C_s = 3C_v$

is stationary. The calculations of the probability of the given filling of the storage volume at the end of a year must take into consideration the state at the beginning of a year, the size of the inflow and withdrawal.

The required distribution of the probability of storage capacity filling $V = f(P)$ that is continuous is approximated by a discrete one by dividing the given size of the storage volume into $n - 2$ equal parts and use their mean values; we add another two values: $V_1 = 0$, expressing complete emptying, and V_n, which is the complete filling of the storage capacity (Fig. 5.9). Each interval ΔV_m has a probability of occurrence ΔP_m and the value V_m has the probability of exceedance P_m.

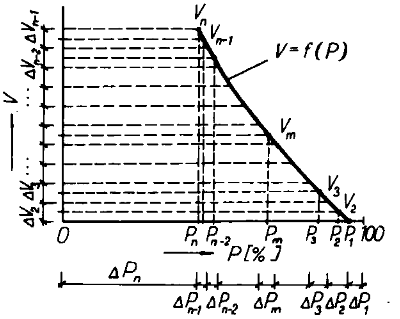

Fig. 5.9 Solution of the probability of the storage volumes (Moran's method)

The probability of exceeding a certain volume V_j is a *conditional probability* including the probability of occurrence of initial filling and also the probability of occurrence of inflow Q_i; the withdrawal is presumed to be constant. For all values $V_1, V_2, \ldots, V_j, \ldots, V_n$ the probability of exceeding $P(V_j)$ can be expressed by the equations

$$\begin{aligned}
P(V_1) &= a_{11}\,\Delta P_1 + a_{12}\,\Delta P_2 + \ldots + a_{1k}\,\Delta P_k + \ldots + a_{1n}\,\Delta P_n \\
P(V_2) &= a_{21}\,\Delta P_1 + a_{22}\,\Delta P_2 + \ldots + a_{2k}\,\Delta P_k + \ldots + a_{2n}\,\Delta P_n \\
&\vdots \\
P(V_j) &= a_{j1}\,\Delta P_1 + a_{j2}\,\Delta P_2 + \ldots + a_{jk}\,\Delta P_k + \ldots + a_{jn}\,\Delta P_n \\
&\vdots \\
P(V_n) &= a_{n1}\,\Delta P_1 + a_{n2}\,\Delta P_2 + \ldots + a_{nk}\,\Delta P_k + \ldots + a_{nn}\,\Delta P_n
\end{aligned} \tag{5.13}$$

where the unknowns are the probabilities of occurrence $\Delta P_1, \Delta P_2, \ldots, \Delta P_n$ and coefficients a_{jk} $(j, k = 1, 2, \ldots, n)$ are conditional probabilities of exceeding the water volume accumulated in the storage capacity at the turn of two years;

$$\text{coefficient matrix } \boldsymbol{A} = \begin{Vmatrix} a_{11}, & a_{12}, & \ldots, & a_{1n} \\ a_{21}, & a_{22}, & \ldots, & a_{2n} \\ \vdots & & & \\ a_{n1}, & a_{n2}, & \ldots, & a_{nn} \end{Vmatrix}$$

is denoted as the *matrix of transient probabilities*, i.e., a transition matrix.

Figure 5.10 shows how to determine a_{jk}.

We construct an exceedance curve $k_r = f(p)$; on the axis of ordinates we put β_z^v and $(\beta_z^v + \alpha)$.

We divide the ordinate β_z^v into $(n - 2)$ equal intervals $\Delta\beta$ (usually 15 to 20 is sufficient) and in each interval we determine, in the centre of the interval, the ordinate β_k.

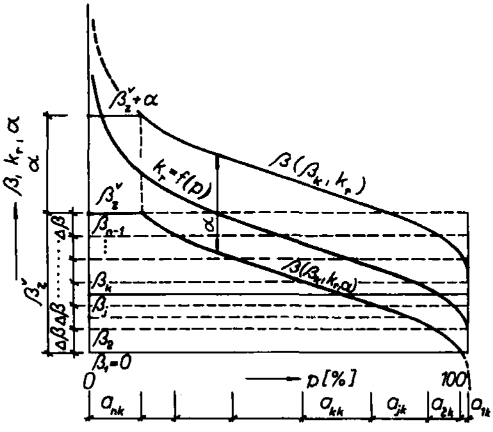

Fig. 5.10 Procedure for determining the matrix of transitional probability coefficients (Moran's method)

If we add to any of the values β_k the ordinates of the curves of exceeding $k_r = f(p)$ and if we bound the thus gained curve by ordinates $(\beta_z^v + \alpha)$ and $\beta_1 = 0$, we obtain a conditional curve of exceeding water supplies prior to withdrawal. After subtracting ordinate α from this curve, we obtain the conditional curve of exceedance after withdrawal β $(\beta_k, k_r, \alpha) = f(p)$; the points of intersection with the boundaries of intervals $\Delta\beta$ then determine intervals on the axis of the abscissas which are the conditional probabilities $a_{1k}, a_{2k}, \ldots, a_{nk}$ sought.

The following relationship is valid between the values of ΔP_k

$$\sum_{k=1}^{n} \Delta P_k = 1 \tag{5.15}$$

The relationship between the probabilities of occurrence and the probabilities of exceedance (illustrated in Fig. 5.9) is

$$\begin{aligned}
P(V_1) &= \Delta P_2 + \Delta P_3 + \ldots + \Delta P_k + \ldots + \Delta P_{(n-1)} + \Delta P_n \\
P(V_2) &= \tfrac{1}{2}\Delta P_2 + \Delta P_3 + \ldots + \Delta P_k + \ldots + \Delta P_{(n-1)} + \Delta P_n \\
&\vdots \\
P(V_k) &= \tfrac{1}{2}\Delta P_k + \ldots + \Delta P_{(n-1)} + \Delta P_n \qquad (5.16) \\
&\vdots \\
P(V_{n-1}) &= \tfrac{1}{2}\Delta P_{(n-1)}) + \Delta P_n \\
P(V_n) &= \Delta P_n
\end{aligned}$$

From equation (5.15) it follows that

$$P(V_1) = 1 - \Delta P_1 \tag{5.17}$$

which brings us to the final equations

$$\begin{aligned}
&(a_{11} + 1)\,\Delta P_1 + a_{12}\,\Delta P_2 + \dots + a_{1k}\,\Delta P_k + \dots + a_{1,n}\,\Delta P_n = 1\\
&a_{21}\,\Delta P_1 + (a_{22} - 0.5)\,\Delta P_2 + \dots + (a_{2k} - 1)\,\Delta P_k + \dots + (a_{2n} - 1)\,\Delta P_n = 0\\
&\vdots\\
&a_{k1}\,\Delta P_1 + a_{k2}\,\Delta P_2 + \dots + (a_{kk} - 0.5)\,\Delta P_k + \dots + (a_{kn} - 1)\,\Delta P_n = 0\\
&\vdots\\
&a_{(n-1)}\Delta P_1 + a_{(n-1)2}\,\Delta P_2 + \dots + a_{(n-1)k}\Delta P_k + (a_{(n-1)(n-1)} - 0.5)\,\Delta P_{(n-1)} +\\
&\qquad\qquad + (a_{(n-1)} - 1)\,\Delta P_n = 0\\
&a_{n1}\,\Delta P_1 + a_{n2}\,\Delta P_2 + \dots + a_{nk}\,\Delta P_k + \dots + a_{n,(n-1)}\,\Delta P_{(n-1)} + (a_{nn} - 1)\,\Delta P_n = 0
\end{aligned} \tag{5.18}$$

The matrix of coefficients of the unknown variables, which we denote by b_{jk}, can easily be calculated from the transition matrix $\boldsymbol{A}$ by adding matrix $\boldsymbol{C}$:

$$\boldsymbol{B} = \begin{Vmatrix} b_{11}, & b_{12}, & \dots, & b_{1n}\\ b_{21}, & b_{22}, & \dots, & b_{2n}\\ \vdots & & & \\ b_{n1}, & b_{n2}, & \dots, & b_{nn} \end{Vmatrix} = \boldsymbol{A} + \boldsymbol{C} \tag{5.19}$$

where

$$\boldsymbol{C} = \begin{Vmatrix} 1 & 0 & 0 & 0 \dots & 0\\ 0 & -0.5 & -1 & -1 \dots & -1\\ 0 & 0 & -0.5 & -1 \dots & -1\\ \vdots & & & & \\ 0 & 0 & 0 & 0 \dots & -1 \end{Vmatrix} \tag{5.20}$$

The results obtained from equations (5.18) are the probabilities of occurrence ΔP_k of filling of the storage capacity V_k, i.e., a discrete distribution curve. By introducing this into equation (5.16), we determine the exceedance curve $V = f(p)$. The value of the probability of exceedance corresponding to $V_1 = 0$ is the equivalent of the required reliability of withdrawal according to the number of years without failure (occurrence-based reliability).

The calculations can also include the influence of the non-zero correlation function of mean annual discharges (instead of the exceedance curves $k_r = f(p)$, the conditional exceedance curves would have to be included in the calculations), as well as the variable withdrawal from year to year (independent or correlationally dependent on the inflow to a reservoir).

5.1.2 Within-year (seasonal) component of the storage capacity

To determine the within-year component V_z^s of the storage capacity the definition given by equation (5.2) should be used as the starting point. According to this, the seasonal component is determined as a certain supplement of the over-year component needed to ensure the reliability which equals the design water supply reliability P; we do not study the reliability p_2 of the seasonal component itself.

The seasonal component V_z^s is therefore the difference between the total size of the storage capacity and the over-year component

$$V_z^s(O_p, p_2) = V_z(O_p, P) - V_z^v(O_p, P) \tag{5.21}$$

or in relative values

$$\beta_z^s(\alpha, p_2) = \beta_z(\alpha, P) - \beta_z^v(\alpha, P) \tag{5.21'}$$

In the hydrological conditions of many rivers, discharge time pattern in the course of a year is very complicated and it is, therefore, impossible according to some authors, e.g., Kritsky and Menkel (1952) or Pleshkov (1961), to apply a simplified schematic division of discharges. To include the variability of discharges within one year is also rather complicated. It is therefore only logical to avoid any complicated methods, such as replacing an analytical (indirect) solution by a direct solution in a synthetic chronological series of mean monthly discharges (see Section 5.2). Modelling of synthetic series gives rise to certain problems, mainly in relation to the

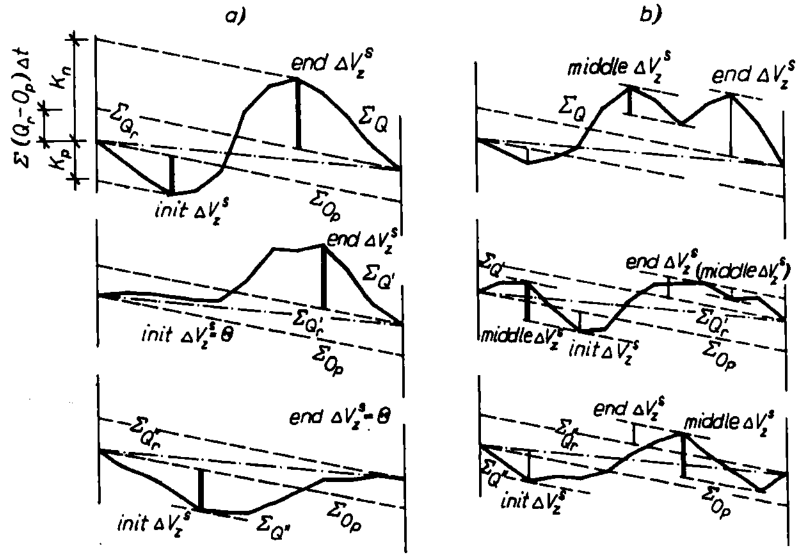

Fig. 5.11 Definition of initial, end and middle part of the seasonal component of the storage capacity (in years with $Q_r \geqq O_p$)

over-year release control, which have not yet been solved satisfactorily. It can be expected that in the future methods based on the division of the storage capacity into over-year and seasonal components will no longer be used.

First we analyse the influence of discharge fluctuations in all the years of the studied discharge series on the size of the storage capacity that is needed to ensure the required withdrawal; however, we must bear in mind that hydrological services usually base their observations on water years, which means that one low-flow period can be divided into two parts.

In the years when $Q_r \geqq O_p$, the over-year component is zero so that $V_z \equiv V_z^s$, or $\beta_z \equiv \beta_z^v$. The seasonal component can be represented by three parts in one hydrological year (Fig. 5.11). In Fig. 5.11a we can find the mass curve of inflow $\sum_Q$ (from a raised pole) in one hydrological year. The parallels to the mass curve of withdrawal $\sum_{O_p}$ determine:

(a) at the beginning of a year, the so-called *initial part of the seasonal component init* ΔV_z^s; its size is given (under the condition that the discharges in a year are expressed, e.g., by mean monthly discharges $Q_{m,i}$) by the largest negative value K_p of the sums

$$V_k = \sum_{i=1}^{i=k} (O_p - Q_{m,i})\,\Delta t_i \tag{5.22}$$

where k has the values $1, 2, \ldots, n$ (n is the number of intervals Δt in a year).

$$init\,\Delta V_z^s = K_p\,(= \max\{V_k\}) \tag{5.23}$$

When all these sums are negative, *init* $\Delta V_z^s = 0$ (Fig. 5.11a);

(b) at the end of year, the so-called *end part of the seasonal component end* ΔV_z^s. For this equation (5.22) is used as for (a), but with the negative extreme, the absolute value of which is denoted by K_n. Then

$$end\,\Delta V_z^s = K_n - \sum_{i=1}^{i=n} (Q_r - O_p)\,\Delta t_i = K_n - 31.6 \cdot 10^6 (Q_r - O_p) \qquad [\mathrm{m}^3] \tag{5.24}$$

Besides those that occur at the beginning or the end of a year, low-flow seasons can occur within a year and can also influence the seasonal component. They can be included in the calculations by determining the size of the so-called *middle part of the seasonal component, mid* ΔV_z^s (Fig. 5.11b) in every year and by adding the values $(O_p - Q_{m,i})\,\Delta t_i$. Then the following conditions have to be fulfilled:

- the *mid* ΔV_z^s can only be calculated after at least one negative value $(O_p - Q_{m,i})$ since the beginning of a year has been determined, which eliminates the possibility of confusing the initial and middle parts of the seasonal component;
- when the added values $\sum (O_p - Q_{m,i})\,\Delta t_i$ reach a negative value, zero is considered;

– the maximum value $\sum(Q_p - Q_{m,i})\,\Delta t_i$ may not occur at the end of a year. Should this be the case, then *mid* ΔV_z^s is considered as the second maximum (local) of the sum $\sum(O_p - Q_{m,i})\,\Delta t_i$; if it does not exist, then *mid* $\Delta V_z^s = 0$ is used;

– for every year only one value of *mid* ΔV_z^s is chosen, given by the maximum $\sum(O_p - Q_{m,i})\,\Delta t_i$, although several low-flow periods might occur in one year.

After determining *init* ΔV_z^s, *end* ΔV_z^s and *mid* ΔV_z^s in every year in which $Q_r \geqq O_p$, we can add the end component of one year and the initial component of the following year [*end* $\Delta V_z^s(t)$ + *init* $\Delta V_z^s(t+1)$], disregarding in the chronological series the years with $Q_r < O_p$. Thus, we obtain a set of values, from which, after ranking in an ascending order and adding the probability, we gain an *empirical probability (reliability) curve* $V_z^s(1) = V_z(1) = f(p)$ reflecting the size of the seasonal component of the storage capacity in the years with $Q_r \geqq O_p$.

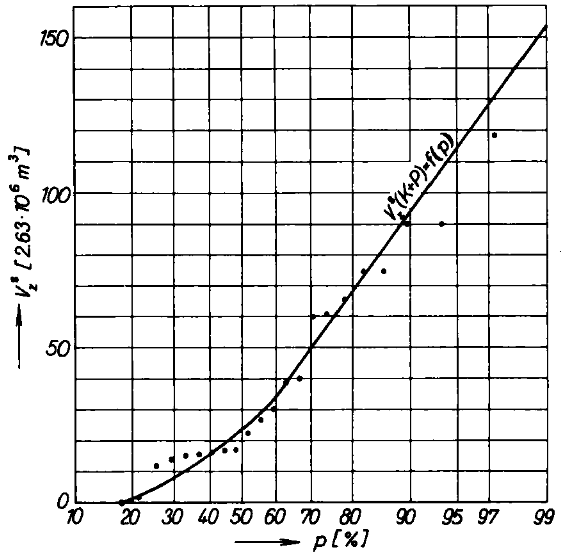

Fig. 5.12 Probability (reliability) curve of the seasonal component V_z^s $(K + P)$, constructed in the probability paper of normal distribution

From the *mid* ΔV_z^s values we can also construct a probability (reliability) curve of *mid* $\Delta V_z^s = f(p)$ which we use to correct, if necessary, the results in the final stage of the calculations.

In constructing probability curves $V_z^s(1) = f(p)$ or *mid* $\Delta V_z^s = f(p)$, the special character of the initial values $V_z^s(1)$ or *mid* ΔV_z^s must be considered, as usually several terms are zero.

These values cannot be fitted by a theoretical distribution function according to some of the methods usually used (moments or quantiles). Therefore, we must draw the empirical probability curve through the observation points plotted on the probability paper (usually of a normal distribution—Fig. 5.12).

Certain experience is needed, especially in those cases where the "empirical" points are not grouped around a curve of a simple type, or when the extreme values deviate from the tendency of increased demand with increased reliability p. Sometimes it is sufficient to eliminate the doubts concerning the pattern of the probability curve $V_z^s(1) = f(p)$ to change the scale of ordinates.

All these circumstances render a practical application very difficult and make it almost impossible to solve the problem with the help of a computer.

Even more complicated is the question of including the *influence of the seasonal component on the size of the storage capacity in over-year low-flow periods.*

Let us presume that before the first and after the last year of a low-flow period (in which $Q_r < O_p$ for all the years) the same probability applies to any of the years with $Q_r \geqq O_p$. Let us therefore consider the sequence of the mean annual discharges as a completely random sequence.

Figure 5.13 illustrates the various possible influences of the fluctuation of discharges within a year on that part of the seasonal component relevant to the limit of the over-year low-flow period. In Fig. 5.13a the value *end* ΔV_z^s of year i, which precedes the low-flow period, is the significant value for the seasonal component. However, a case might occur when the size of the seasonal component is determined by the changes of discharges in the year $(i - 1)$—see Fig. 5.13b, or $(i - 2)$, etc.; the probability of such a state decreases with increasing distance from the low-flow period.

The discharge fluctuations in the years of the low-flow periods $(Q_r < O_p)$ can also determine the size of the seasonal component. Figure 5.13c illustrates a case where the

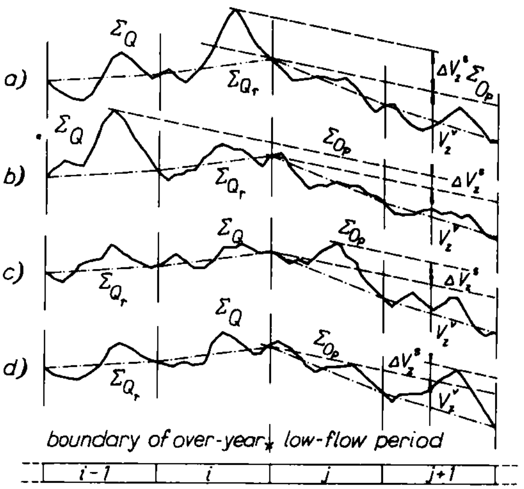

Fig. 5.13 Influence of discharge fluctuations in years near boundary of an over-year low-flow period on the size of the seasonal component of the storage capacity

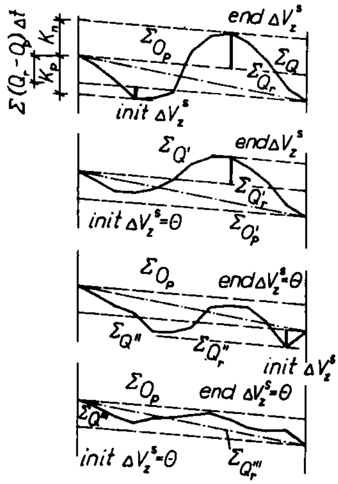

Fig. 5.14 Definition of initial and end part of seasonal component in low-flow years $(Q_r < O_p)$

size of part of the seasonal component at the beginning of the low-flow period is given by the value *end* ΔV_z^s of year j, which is the first year of the over-year low-flow period; however, the year $(j + 1)$ can also be decisive—see Fig. 5.13d, $(j + 2)$, etc. Similar conditions can occur at the terminal boundary of the low-flow period.

As we have proved, the influence of the years $Q_r < O_p$ on the size of the seasonal component, the initial and end parts of the seasonal component in those years (Fig. 5.14), must be defined. Similarly, as in the years with $Q_r \geqq O_p$ the mass discharge curve $\sum_Q$ (with a raised pole) is illustrated and parallels with the mass curve of withdrawal $\sum_{O_p}$ are drawn in the terminal points at the turn of the year. The vertical distance between those parallels illustrates the deficit between the annual volumes of outflow and of inflow, which means that it is a part of the over-year component of the storage capacity.

The initial part of the seasonal component must ensure an augmented discharge in the low-flow periods at the beginning of a water year; however, only the part that exceeds the deficit of the annual volume of inflow (Fig. 5.14). The same applies to the end of a water year.

If we use discrete values of $Q_{m,i}$ for the discharge distribution within a year, we add, similarly to the case for years with $Q_r \geqq O_p$

$$V_k = \sum_{i=1}^{i=k} (O_p - Q_{m,i})\,\Delta t_i$$

where k has the value $1, 2, \ldots, n$ (n is the number of discrete values of $Q_{m,i}$, or intervals Δt_i in a year).

We take the largest value of the positive sum (denoted by K_p) and the negative extreme (if it exists), the absolute value of which we denote by K_n. Then

$$\mathit{init}\,\Delta V_z^s = K_p + \sum_{i=1}^{i=n} (Q_r - O_p)\,\Delta t_i = K_p + 31.6 \cdot 10^6 (Q_r - O_p) \tag{5.25}$$

$$\mathit{end}\,\Delta V_z^s = K_n \tag{5.26}$$

If all values $\sum_{i=1}^{i=n} (O_p - Q_{m,i})\,\Delta t_i$ are positive, *end* $\Delta V_z^s = 0$.

That these algebraic terms are true can be seen from Fig. 5.14, which also illustrates the discharge distribution within a year, where the *init* ΔV_z^s or *end* ΔV_z^s, or both parts of the seasonal component are zero ($\sum_{Q'}, \sum_{Q''}, \sum_{Q'''}$).

First we include the influence of the years with $Q_r \geqq O_p$, preceding or following an over-year low-flow period in the seasonal component. For the directly preceding to following years the values are *end* ΔV_z^s (of the preceding year) and *init* ΔV_z^s (of the following year) determined by equations (5.23) and (5.24). As there is always at least one low-flow year between the corresponding values *end* ΔV_z^s and *init* ΔV_z^s, they can be considered independent. Therefore (from empirical values), probability curves *init* $\Delta V_z = f(p)$ and *end* $\Delta V_z = f(p)$ are constructed and by their composition the probability curve $V_z^s(i) = f(p)$ is obtained, which expresses the influence of the years directly preceding or following the low-flow period on the size of the seasonal component.

The way to construct the probability curves by composition can be seen in Fig. 5.15. In suitably selected intervals one of the probability curves is replaced by mean values and to these a second probability curve is "added", with the condition that the width of every interval is 0–100%. By adding the horizontal partial coordinates of the thus-constructed probability curves for the selected values of the ordinates, the resulting probability curve is obtained.

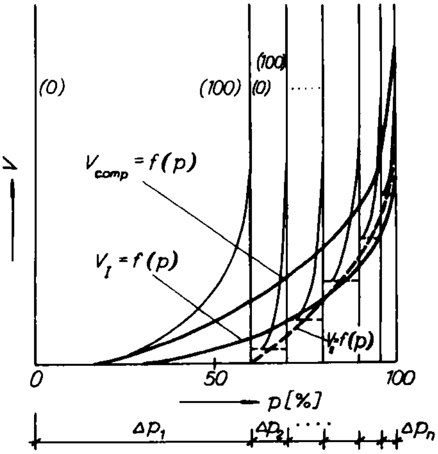

Fig. 5.15 Diagram of the composition of probability curves (presuming mutual independence of the members of the two samples)

In considering the influence of the years more distant from the low-flow year, it should not be overlooked that between the year under observation and the first low-flow year there are x years with $Q_r \geqq O_p$. Then for the seasonal component of the $(i - x)$th year

$$V_z^s(i - x) = V_z^s(i) - \sum_{i-x}^{i} (Q_r - O_p)\, T_r \tag{5.27}$$

where T_r is the number of seconds per year $(31.6 \cdot 10^6)$.

From the probability curves $V_z^s(i)$, $V_z^s(i - 1)$, etc., those values must be chosen which, in every probability sphere, have a larger seasonal component V_z^s and are therefore significant values for the design of the storage capacity. The significant values are determined by dividing one of the probability curves (I) into intervals Δp, in which it is replaced by mean values. In each interval from 0–100% a second probability curve (II) is drawn, which by its point of intersection with the mean value of the first curve (Fig. 5.16) divides every interval Δp into a part $\Delta p'$ in which the values V_z^s of the first curve are larger than those of the second curve and therefore are significant values, and into a part $\Delta p''$, with the opposite conditions. The segments of the first probability curve in the respective intervals are drawn by a linear reduction of the abscissa into intervals $\Delta p'$, where they are significant, while in the intervals $\Delta p''$ the segments of the second probability curve are valid (as they are larger). By joining the horizontal coordinates of the probability curves in all partial intervals for the chosen ordinates V_z^s, we obtain the required probability curve.

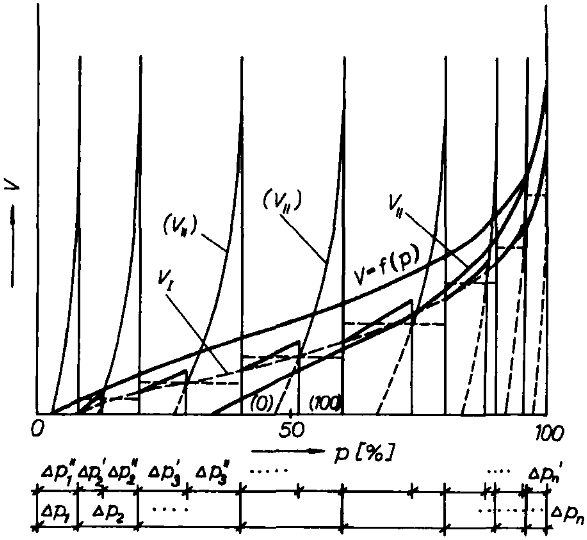

Fig. 5.16 Diagram of the choice of the significant values of two probability curves

This method brings us to a single probability curve $V_z^s\,(i, i-1, i-2, \ldots) = f(p)$ reflecting the influence of the years with $Q_r \geqq O_p$ on the size of the seasonal component in over-year low-flow periods.

We proceed similarly for years with $Q_r < O_p$. From the probability curves *init* ΔV_z^s and *end* ΔV_z^s, curve $V_z^s(j) = f(p)$ is obtained, expressing the influence of the years directly preceding or following the low-flow period.

For years more distant from the low-flow period it follows that

$$V_z^s(j + y) = V_z^s(j) - \sum_{j}^{j+y} (O_p - Q_r)\, T_r \tag{5.28}$$

By selecting significant values from the probability curves $V_z^s(j)$, $V_z^s(j + 1), \ldots$ (see Fig. 5.16) the resulting probability curve $V_z^s(j, j + 1, \ldots) = f(p)$ is obtained, expressing the influence of the years with $Q_r < O_p$ on the size of the seasonal component.

As on the boundary of an over-year low-flow period, one group of years with $Q_r \geqq O_p$ meets a group of years with $Q_r < O_p$, the significant value (i.e., larger) must be chosen, regardless of the group of years to which it belongs.

We therefore choose significant values in the probability curves $V_z^s(i, i - 1, \ldots)$ and $V_z^s(j, j + 1, \ldots)$ (see Fig 5.17) and consider the width of the probability space to be the same (one value of the first group of years always meets one value of the second group, even when the number of years in each group varies, i.e., the width of the intervals $\langle 0; p_\alpha \rangle$ and $\langle p_\alpha; 100\% \rangle$ is different). The result is a probability curve $V_z^s(2) = f(p)$ which is an expression of the *seasonal component of a reservoir storage capacity in over-year low-flow periods.*

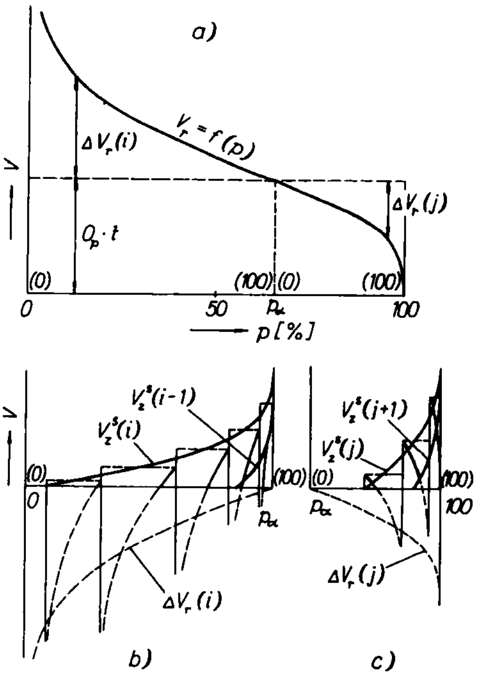

Fig. 5.17 Determination of probability curves reflecting the influence of the years more distant from the boundary of the over-year low-flow period on the size of the seasonal component

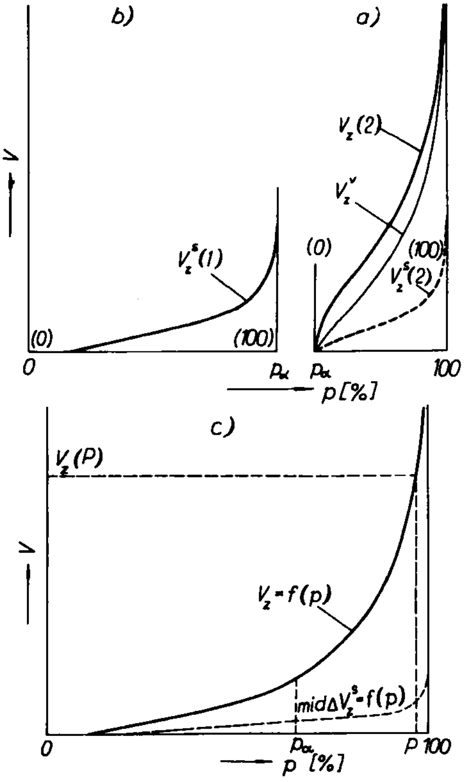

Fig. 5.18 Construction of the resultant storage capacity probability curve $V_z = f(p)$

It is also necessary to consider the *over-year component* V_z^v. It can be presumed that the two quantities (V_z^v, V_z^s) are independent and we therefore obtain the resultant probability curve $V_z(2) = f(p)$ by the composition of the curves V_z^v and V_z^s (Fig. 5.18a). Curve $V_z^v = f(p)$ can best be constructed with the help of a nomograph (see Section 5.1) for p_α, $V_z^v = 0$. As one value of V_z^v is put together with one value of V_z^s, the width of the probability of both curves is identical.

Curve $V_z(2)$ determines the *demand of the over-year low-flow periods on the size of the storage volume*; from the whole probability of exceeding mean annual discharges it refers to the interval $\langle p_\alpha; 100\% \rangle$ in which $Q_r < O_p$ (Figs. 5.17, 5.18). Curve $V_z(1) = f(p)$ is valid in the interval $\langle 0; p_\alpha \rangle$ where $Q_r \geq O_p$ (Fig. 5.18b). As the storage capacity can either provide the yield in a relatively deep discharge depression of a low-flow season in one single year, or cover over-year low-flow seasons (with a smaller discharge decrease), we put together the two probability curves simply by adding the horizontal coordinates and then obtain curve $V_z = f(p)$—Fig. 5.18c.

Finally, it is possible to take into consideration the influence of low-flow seasons within the respective years, expressed by the probability curve $mid\,\Delta V_z^s = f(p)$, and by selecting significant values to correct the curve $V_z = f(p)$. This step, however, is not frequently taken as the curve $mid\,\Delta V_z^s$ usually does not influnce the shape of curve V_z in the sphere of higher probability values.

The theoretical size of the storage capacity $V_z(p)$ sought is thus determined for the given design reliability P (according to the number of years without failure).

This method is so far the most perfect for the calculation of the seasonal component of the storage volume and is an original Czech contribution (Klemeš, 1963). As it is methodologically as well as technically very demanding, it is not yet widely used in practice. It can be found in greater detail in other studies (e.g., Votruba and Broža, 1974).

This method can also be used in those cases where the value O_p changes during the year, but where the withdrawal diagram is the same in all the years. The over-year component is then determined for a mean withdrawal $\bar{O}_p$; the variability of withdrawal has to be included in the calculations of the respective parts of the seasonal component.

However, the method can be simplified. It is, e.g., possible to disregard the influence of the years $(i - 1)$ and $(j + 1)$ on the size of the seasonal component.

With a high relative yield $\alpha = O_p/Q_a$ $(\alpha > 0.7)$, the probability curve $V_z(1)$ in the interval of the seasonal release $\langle 0; p_\alpha \rangle$ need not be constructed as it does not influence the shape of curve $V_z = f(p)$ in the sphere of high values of p (Fig. 5.18); the resultant value $V_z(P)$ can then be read directly from the probability curve $V_z(2)$.

Another simplification which, however, can make the solution less accurate, is to disregard the influence of the years with $Q_r < O_p$ on the seasonal component.

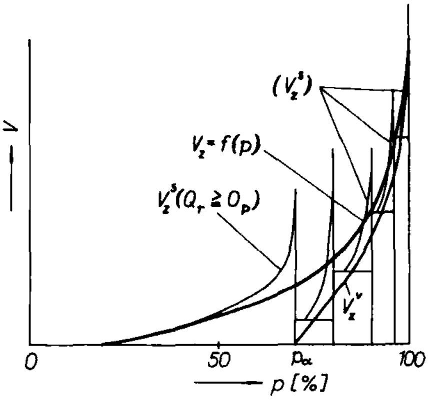

Fig. 5.19 Diagram of the simplified method of determining the storage capacity probability curve (Andreianov, 1960)

Then the probability curve $V_z^s(2)$ is identical to curve $V_z^s(i)$. This simplification is very similar to *Andreianov's method* (1960), which is based on the so-called water management years, which make it possible to determine the seasonal components in the respective years as a whole; considering, however, only years with $Q_r \geqq O_p$. It is presumed that the probability curve $V_z^s = f(p)$, which is very close to curve $V_z^s(1)$ mentioned in the original method, is valid both in the interval $\langle 0; p_\alpha \rangle$ and in the interval of low-flow years $(p_\alpha; 100\%)$ where it meets the over-year component V_z^v (Fig. 5.19). Despite its methodological shortcomings, Andreianov's method is satisfactory from the point of view of accuracy, especially with a high relative yield (α).

5.2 METHOD BASED ON SYNTHETIC DISCHARGE SERIES

Reservoirs designs based on simulation in time discharge series (Chap. 3) are simple, and make it possible to solve cases of complicated withdrawal (release) diagrams. "Classical" method determined the parameters of discharge control α and β_z (or O_p and V_z) in real time series. The size of the storage capacity was therefore given by one single series of discharge conditions in the past which will not be repeated during the operation of a reservoir (i.e., in the next few decades).

Even during relatively long observations (40 to 50 years), in real discharge series only a very few low-flow periods lasting longer than one year occur which could greatly influence the size of the storage capacity (on the average one low-flow period in 10 to 12 years). Therefore, the shortest permissible length of time series in relation to the type of release control was recommended. In the conditions pertaining, e.g., in Middle Europe, such a recommendation is rather risky (Votruba and Broža, 1966); for over-year release control it is therefore necessary to apply, for the determination of the parameters of release control (α, β_z, P), *synthetic chronological discharge series* (also called *random*, *synthetic*) of a sufficient length (as a rule 500–1000 years; Broža, 1973).

In preliminary studies for the comparison of various alternatives (mainly from the qualitative point of view), solutions in real series cannot be excluded.

A condition is a given withdrawal (release) from a reservoir in every step of the solution. Let us consider a long random discharge series, in which every member $Q(\Delta t_i)$ is an expression of the mean discharge in the time interval Δt_i; the required release (withdrawal) from the reservoir is given at each interval Δt_i by the mean value $O_p(\Delta t_i)$.

The *balance deficit* $\Delta V(\Delta t_i)$ in the interval Δt_i is given by

$$\Delta V(\Delta t_i) = [O_p(\Delta t_i) - Q(\Delta t_i)]\,\Delta t_i \tag{5.29}$$

In the problem, the withdrawal O_p (i.e., α) and the size of the storage capacity V_z' (i.e., β_z') are given; the reliability of water supply p is to be calculated.

At the beginning of the series, the first filling of the storage volume has to be chosen. With a sufficiently long series, the choice of the first filling has practically no influence on the result. Usually the storage capacity is considered to be full.

Additions are made

$$V_k = \sum_{i=1}^{i=k} \Delta V(\Delta t_i) = \sum_{i=1}^{i=k} [O_p(\Delta t_i) - Q(\Delta t_i)] \, \Delta t_i \tag{5.30}$$

with the following limiting conditions (Fig. 5.20):

(a) if $V_k < 0$, $V_k = 0$ is considered as the result in this step, and this value is used to calculate V_{k+1}, V_{k+2}, etc.;

(b) if the value of $V_k > V_z'$, the result of the calculations in the given step is $V_k = V_z'$, with the respective consequences in the further steps ($k + 1, \ldots$).

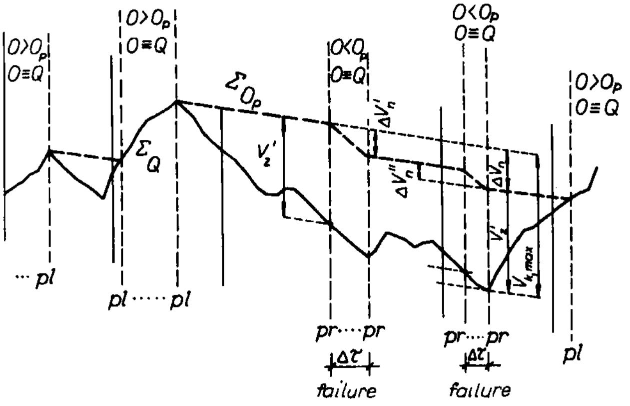

Fig. 5.20 Simulation of the work regime of a reservoir by means of inflow and release mass curves

The first condition reflects the state with a full storage capacity, when the surplus inflow to a reservoir (above the value O_p, or any other real release value) cannot be accumulated, causing an excess outflow. The second condition reflects the emptying of the active storage capacity. If the inflow to a reservoir continues to be less than the release (withdrawal), then a failure to supply water occurs as $O \equiv Q < O_p$.

To express the reliability of water supply in numbers according to various aspects, the following must be recorded:

1. number of years (water years), in which failures to supply water occur,
2. duration of failures to supply water $\Delta\tau$, when $V_k = V_z'$ and $Q < O_p$,
3. volume of water deficits with respect to the planned withdrawal, given by $\Delta V_n = \sum[O_p(\Delta t_i) - Q(\Delta t_i)] \, \Delta t_i$, starting with the moment when $V_k \geqq V_z'$ up to the moment when $Q \geqq O_p$ (Fig. 5.20).

It is also expedient to know the maximum deficit in every case; this is given by the difference $\Delta O = (O_p - Q)_{\max}$ while at the same time fulfilling $V_k = V_z'$ (the active storage capacity is empty).

From these data it is possible to express the reliability of water supply according to the number of years without failure (occurrence-based reliability).

$$p_o = \frac{N - n}{N} 100 \quad [\%] \tag{5.31}$$

where n is the number of years from the whole series in which failure to supply water occurs, and N is the total number of years of a synthetic series.

With a sufficiently long series (500–1000 years) the differences as compared with the usual relationship $p_o = 100[(N - n) - 0.3]/(N + 0.4)$ [%] are negligible.

The time-based reliability (of water supply according to duration) p_t is determined with regard to the length of the series with a duration T (in units of time)

$$p_t = \frac{T - \sum \Delta\tau}{T} 100 \quad [\%] \tag{5.32}$$

where $\sum \Delta\tau$ is the sum of the duration of all failures that occurred in the series.

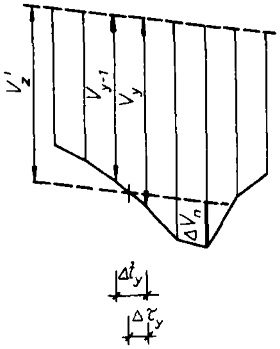

Fig. 5.21 Linear interpolation with time-based reliability

The failure usually starts somewhere in the middle of the interval Δt_y at the beginning of which the value of the emptying of the storage value is $V_{y-1} < V_z'$, while at the end it is $V_y > V_z'$; therefore part of interval Δt_y is usually considered as the failure part and is denoted by $\Delta\tau_y$, resulting from linear interpolation (Fig. 5.21)

$$\Delta\tau_y = \frac{V_y - V_z'}{V_y - V_{y-1}} \Delta t_y = \frac{V_y - V_z'}{[O_p(\Delta t_y) - Q(\Delta t_y)]} \tag{5.33}$$

In calculating the quantity-based reliability (or withdrawal according to the volume of the supplied water) p_d, the theoretical volume of supplied water without any failure for the whole period, i.e., for the whole random series must first of all be determined

$$V_d = \sum_{i=1}^{n} O_p(\Delta t_i)\, \Delta t_i \tag{5.34}$$

The value of p_d is calculated from the equation

$$p_\mathrm{d} = \frac{V_\mathrm{d} - \sum \Delta V_\mathrm{n}}{V_\mathrm{d}} 100 \quad [\%] \tag{5.35}$$

In turn, further values of V_z' (or β_z') are chosen while preserving the rates of O_p (or α) and the calculations are repeated. Thus, the relationship $V_z' = f(p)$ or $\beta_z' = f(p)$ is constructed and for the design value of reliability P, the size of the storage capacity is subtracted (Fig. 5.22).

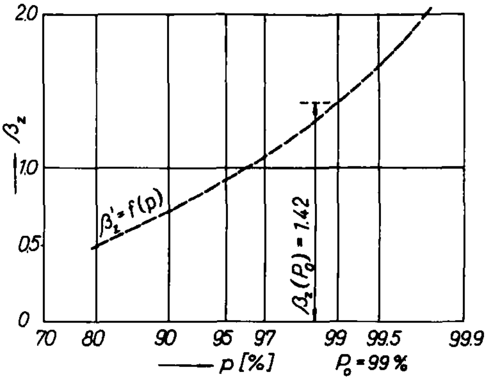

Fig. 5.22 Determination of the size of the storage capacity by means of the storage capacity probability curve (relationship constructed by repeated simulation)

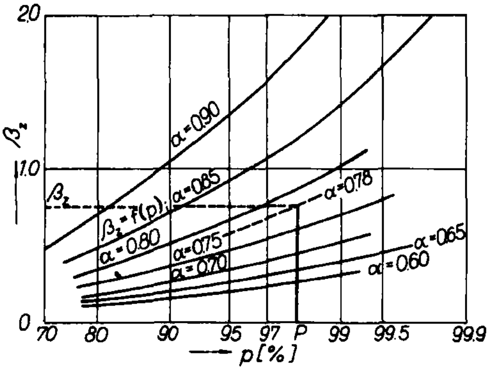

Fig. 5.23 Relationship between the basic characteristics of the reservoir storage function (P, β_z, α) in terms of the results of the simulation in synthetic series (by computer)

The general algorithm of a direct solution could be perfected by applying an iteration method to a computer and thus obtaining the design value of the storage capacity V_z directly; however, a more rational method is that which leads to the relationship $V_z = f(p)$.

A rough idea about the size of the storage capacity is obtained by calculating the volume V_z^m, which in the synthetic series ensures the required water supply with a 100% reliability. One proceeds again according to equation (5.30), however, without the limiting condition $V_k \leqq V_z'$. The size of volume V_z^m equals the absolute maximum from among the values V_k.

If the size of the storage capacity V_z (or β_z) and the design reliability P are given and calculations are to determine the size O_p, reliability p must be determined for several selected values of O_p' (with a given V_z), which makes it possible to construct the relationship $O_p' = f(p)$ (or $\alpha' = f(p)$), where for the design reliability P the sought size of the yield (withdrawal) O_p must be found.

The relationships between the characteristics of release control can be expressed with the help of three-dimensional graphs (P, V_z, O_p) or (P, β_z, α) (Fig. 5.23). For various suitably selected values of O'_p (or α'), the relationship $V'_z = f(p)$ [or $\beta'_z = f(p)$] is constructed. The vertical sections of this graph ($P = \text{const}$) are the storage-yield curves $\beta_z = f(\alpha)$ or $V_z = f(O_p)$.

5.2.1 *Over-year component of the storage capacity*

Synthetic series of mean annual discharges are usually not used for practical calculations of the over-year component of the storage capacity as there are simpler methods. However, as they were frequently used for calculations of a research nature we shall mention this method briefly as a special case of a general algorithm of simulation calculations.

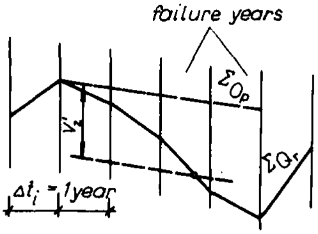

Fig. 5.24 Failure years in synthetic series of mean annual discharges using the simulation method

In a random series of mean annual discharges ($\Delta t_i = 1$ year), only the reliability from the point of view of years without failure (occurrence-based reliability) is significant, as the duration and extent of the shortages are greatly distorted. Therefore, for direct calculations with the given values O_p and V'_z or α and β'_z, only the number of years in which failures to supply water occur are studied. Considered as failure years are those years in the course of which the reservoir volume V'_z is emptied (Fig. 5.24). The sought reliability p_0 is determined from equation (5.31). It is possible to determine the size $V_z(P_0)$ with a given value O_p, which ensures a water supply with the selected (required) reliability P_0, by approximation (selecting various V'_z).

Some authors used this method instead of the analytical method to construct graphs for a quick determination of the over-year component of a storage capacity (see Section 5.1). The reason is that modelling of synthetic series (including indispensible testing), and the actual solution as well as the smoothing of the results can be carried out completely by a digital computer, whereas some of the operations included in the analytical calculations cannot be easily programmed and carried out by computer. If we consider that, with regard to the general character of this method, various values of C_v, C_s, $r(\tau)$ or their combinations have to be chosen for modelling the series and that for the testing a large number of identical realizations of the random series (e.g., ten) are needed, it is obvious that many calculations have to be made to construct the nomograms $\beta^v_z = f(C_v)$ for various values of α and for P_0, C_s and $r(\tau)$. Computers are able to handle this problem as a whole and therefore graphs constructed in this way are much more universal than Pleshkov's or Gugli's graphs.

Among the first persons to use synthetic series was Svanidze (1964), who worked with series of mean annual discharges of 1000 years; similar graphs were elaborated by Reznikovski (1969) and Nacházel (1973).

Direct calculations in synthetic series of mean annual discharges were further used, e.g., in the study of non-stationary discharge series and various types of their models (Nacházel and Patera, 1975).

5.2.2 Total size of the storage capacity

The lack of analytical methods, i.e., the necessity to divide the total size of the storage capacity into the over-year and the seasonal components, led to research on methods which would include both the over-year and the seasonal cycle of a reservoir. The introduction of matrix methods (Moran et al., and especially Lloyd (1963)), in which the inflow to a reservoir is considered to be a simple Markov chain, represented a great progress in this respect. However, it was only by the elaboration of mathematical models of long discharge series, including the *fluctuations of discharges within the respective years*, that a qualitatively higher level was reached in the calculations of reservoir regulation.

Synthetic series of mean monthly discharges are usually sufficiently detailed for over-year release control. The error introduced by the gross character of these data is permissible and can be compensated by a certain percentage increase of the theoretical size of the storage capacity (Section 5.3), or it can be neglected.

In synthetic series of mean monthly discharges, algorithms described in the introduction to Section 5.2 are used, i.e., according to equations (5.29) to (5.35). If we study only the reliability, based on occurrence, the results of the calculations are equivalent to the results of the analytical method (using suitable graphs to determine the over-year component and a detailed solution of the seasonal component). However, from a long synthetic series further information can be gained making it possible, e.g., to analyse the failure periods, to determine the rules for reservoir operations, etc.

The design reliability of water supply reflects the probability with which the consumer's demands will be met over a long period; these are characteristics which cannot reflect the real conditions in the next several decades for which we are able to predict the demands on a reservoir. A method should therefore be used in which a long synthetic series is replaced by a *set of short realizations of synthetic series of mean monthly discharges*. Each such series can be considered to be one of the possible discharge series in the future reservoir operations (with the same probability of occurrence). In each series (e.g., of 50 years) direct calculations are carried out which lead to a set of results (e.g., the size of the storage capacity). Further statistical processing leads to the design size of the studied characteristics of release control.

In Fig. 5.25 there are plotted the points of the "empirical" curve of exceeding the relative sizes of storage volumes β_z, necessary to ensure withdrawal $\alpha = 0.7$ in 50-year realizations of series of mean monthly discharges. The variability of the necessary volumes (with a realibility of 100% in each series) is relatively great, $C_v \doteq 0.21$. When comparing the results gained from shorter random series with the results of 2000-year synthetic series (which were made up of 40 fifty-year series), is was found that the value $\beta_z = 0.358$, which in the long series has a reliability of $p_0 = 95\%$, has a reliability of 12%; similarly with $\beta_z = 0.424$ with a reliability of $p_0 = 97\%$ in the long series, the reliability of the storage volume in 50-year series is 34% (Fig. 5.25).

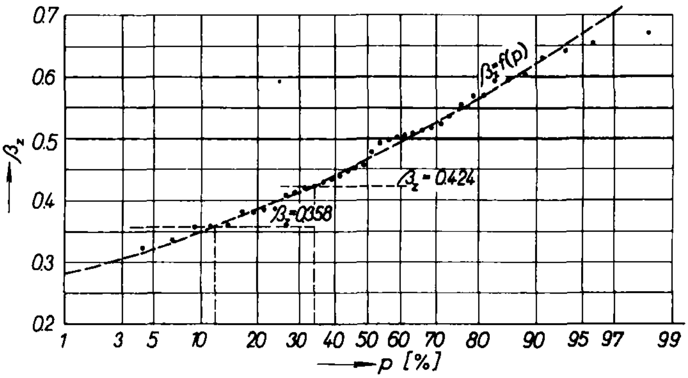

Fig. 5.25 Probability (reliability) curve of storage capacity gained from a sample of 50-year realizations of mean monthly discharge series

The idea to use a set of shorter series instead of one sufficiently long synthetic flow series originated from the need to simulate the possible changes of the hydrological regime in the future reservoir operation. As the analysis of the reservoir function can be done in every short series, it is possible to obtain by this method a set of results reflecting the possible variation of the necessary storage capacities of the reservoir for the given yield and, on the other hand, for the constructed reservoir, the probable variation of the yield in the relation to the hydrological conditions.

From the view of the recent application of the theory of estimation on water resources analysis this method has some drawbacks. These drawbacks are given by the relationship between the sample of statistical characteristics of shorter series (i.e. the realizations of the population) and the parameters of the long series that simulate the population. The results of the research show that this relationship is very complicated as it depends on many factors (e.g. the type of probability distribution of the long series, its autocorrelation, length and number of the shorter series etc.).

These relations can exercise negative impacts on the analysis of the reservoir function in which it is necessary to determine, for the given yield, the design active storage capacity based on the sample of values obtained by the analysis using the shorter series. It was proved that this sample characteristics can be biased under the capacity value derived from the long series and that the results of such analysis can result in underestimating the reservoir storage capacities. The risk of such a fault is the greater, the smaller the set of synthetic series is (i.e. it includes a small number of series) and the synthetic series are shorter.

In the analysis of the inverse task, when the active storage capacity is given and the yield is calculated, a similar risk occurs—there is a tendency to find higher effects (yield) as compared to the analysis based on the long series.

The research activities (Nachâzel, 1987) included further complicated issues—e.g. the relation of the reliability of the yield, based on the set of shorter series and that based on the long series. It this case it was proved that the variation of the reliability values in case of the set of shorter series is extensive and may result in the unjustified increase of the reservoir yield.

The research, therefore, resulted in the statement that the design of reservoir parameters should be based on the hydrological data formed by one sufficiently long synthetic flow series, derived from the representative input parameters. The method using a set of shorter series can be used as a supplementary aid for the estimation of the probable variation of the yield from the designed (constructed) reservoirs for different hydrological conditions.

5.3 EVALUATION OF THE RESULTS OF THEORETICAL CALCULATIONS AND FURTHER INFLUENCES ON THE DESIGN OF THE STORAGE CAPACITY

Theoretical calculations, which determine the basic characteristics of reservoir functions O_p, V_z, P (or α, β_z, P) are the first stage in designing the size of the storage capacity. An important part of these calculations is the evaluation of the results attained and the inclusion of previously not considered influences (e.g., water losses from a reservoir).

An evaluation of theoretical results must consider the following:

(a) reliability of input data, mainly hydrological data and the method used to express the quantity of the withdrawal (release) from a reservoir,

(b) theoretical and practical accuracy of the method used,

(c) possible variability of results caused by random deviations of statistical characteristics of other elements of a real discharge series.

The resultant (design) size of the storage capacity of a reservoir must further include

– demands on the storage capacity to cover water losses, usually caused by evaporation from the water level, in some cases also caused by uncontrollable seepage of water from a reservoir,

– application of "inaccurate" hydrological data resulting from the choice of mean monthly discharges, such as the characteristics of discharge fluctuations during a year,

– reserves which serve to meet various demands not included in the planned storage function (e.g., to flush a river downstream of a reservoir, to influence the temperature of a stream in winter as well as in summer, to alleviate failures of water supply, etc.).

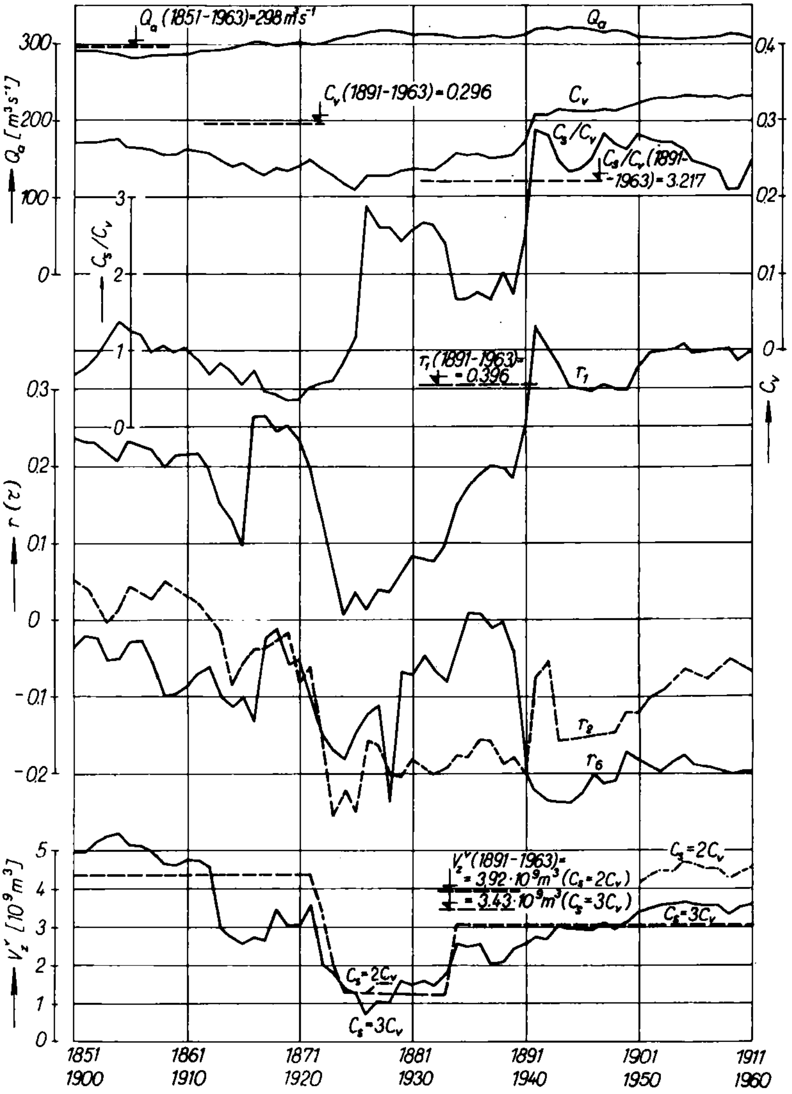

Fig. 5.26 Variability of statistical characteristics and sizes of the over-year component of storage capacity determined from "moving" 50-year discharge series in Děčín on the Labe (samples from the period 1851–1969)

Of great importance for the results of theoretical calculations of the characteristics of an over-year release control is the input real discharge series, especially its length and position in time. Not even probability methods can exclude the direct relationship of the statistical characteristics, correlation relations (or characteristics of the changes in the discharges in the initial real series) and input data of reservoir regulation analysis.

Already in 1963 L. Votruba and V. Broža pointed out the great variability of the statistical characteristics of mean annual discharges of the rivers in the Labe catchment in relation to time and the length of the period of observation. Since then several authors have studied this problem (e.g., Nacházel and Patera, 1975).

Figure 5.26 illustrates the changes in the statistical characteristics Q_a, C_v, C_s/C_v and selected values of the correlation function r_1, r_2, r_6 in "moving" 50-year series from a 110-year series 1851–1960 at the river Labe – Děčín site. Also plotted, are the over-year components of the storage capacity with a yield (release) $O_p = 210\,\mathrm{m^3\,s^{-1}}$ ($\alpha \doteq 0.7$) derived from the statistical characteristics and a correlation function with a reliability of 99%, and given by the direct solution for the respective fifty-year periods (classical method).

The variability of the over-year component of the storage capacity caused by random deviations of statistical characteristics is very great. With regard to the value V_z^v derived from the whole 110-year series, deviations currently exceed ±20% and in some cases even 50%.

Figure 5.26 also shows that there is no direct relationship between the statistical characteristics of discharge in the given period and the demand on the size of the storage capacity in the same period.

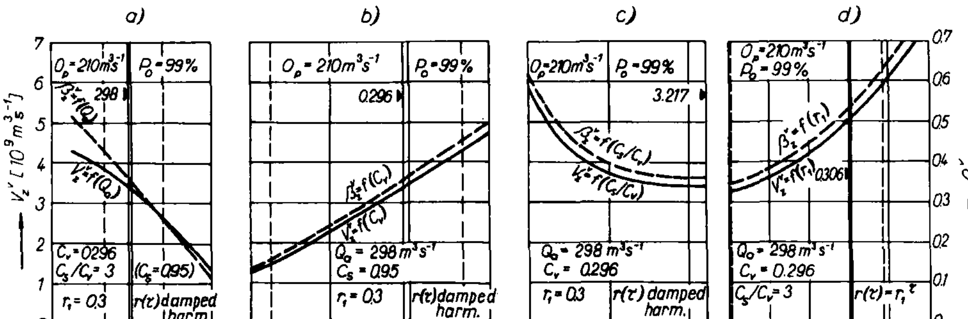

Fig. 5.27 Variability of over-year component of the storage capacity with the change of the respective statistical characteristics and correlation coefficient r_1

In Fig. 5.27 it can be seen how sensitive the over-year component is to any change of one of the characteristics Q_a, C_v, C_s and r_1, while the others remain constant. The statistical characteristics of the 110-year series of the river Labe – Děčín were considered average, deviations were considered in the range of the changes of the "moving" 50-year series.

None of the influences caused by changes of the considered characteristics on the size of V_z^v can be considered to be insignificant. As the result of a change in the mean discharge Q_a (Fig. 5.27a) and with a constant withdrawal O_p, the relative yield α changes (with an increasing Q_a the value of α drops); in this case the relation $\beta_z^v = f(Q_a)$ is not equivalent to the relation $V_z^v = f(Q_a)$.

A change in C_v whilst preserving the value C_s (Fig. 5.27b) automatically leads to a change in the C_s/C_v ratio, which further increases the size V_z^v (or β_z^v) with the increase of C_v. In our case, the influence of the change in C_s/C_v ratio was practically negligible as it moved within the range of 2.7 to 4.7 where the change of ΔV_z^v and the change of $\Delta(C_s/C_v)$ are very small. The influence of the change is C_s/C_v (Fig. 5.27c) is very significant if $C_s/C_v < 2$. This naturally cannot be generalized for higher C_v (above 0.5), see Reznikovski *et al.* (1969).

Table 5.1 Influence of the beginning of observations (length of series) on the characteristics Q_a, C_v, C_s, r_1 and the size of the over-year component of the storage capacity (the river Berounka–Křivoklát)

Period	Duration of series years	Q_a $[m^3 s^{-1}]$	C_v	C_s	C_s/C_v	r_1	V_z^v $[10^6 m^3]$ $O_p = 18 m^3 s^{-1}$	$O_p = 21 m^3 s^{-1}$	$O_p = 24 m^3 s^{-1}$
1951–1970	20	30.9	0.393	0.35	0.89	–	114	338	666
1941–1970	30	31.4	0.464	1.35	2.91	–	179	444	823
1931–1970	40	31.6	0.455	1.25	2.74	0.532	166	417	777
1921–1970	50	31.3	0.440	1.19	2.69	0.475	158	403	769
1911–1970	60	31.2	0.414	1.20	2.90	0.427	126	361	707
1901–1970	70	30.7	0.396	1.34	3.40	0.425	115	352	696
1891–1970	80	30.6	0.379	1.37	3.60	0.411	108	328	657

Usually discharge series are available from the time of the installation of a discharge-gauging site up to the present, and it should therefore be realized what consequences for the estimation of statistical parameters and for the over-year component the beginning of observations can have. Table 5.1 gives the statistical characteristics determined from 20 years, 30 years, ..., 80 years, always ending in 1970, and the size of the over-year component V_z^v for various values of O_p ($P_o = 97\%$, presuming a correlation function in terms of K. Nacházel) on the river Berounka – Křivoklát site. The last 30- as well as 40-year periods have very high C_v values as compared to the 80-year series; coefficients of skewness are always very high. Differences in the values of V_z^v are large. If, for example, the series 1941 to 1970 were used, the values of V_z^v would be larger by 65% with $O_p = 10 m^3 s^{-1}$, by 35% with $O_p = 21 m^3 s^{-1}$ and by 25% with $O_p = 24 m^3 s^{-1}$ as compared with the values of the eighty-year series 1891–1970.

For the discharge series Labe – Děčín and Berounka – Křivoklát statistical characteristics were determined by the method of moments, which is suitable for computer processing, but does not enable a deeper analysis of any anomalies occurring in the observation period. In both series an exceptional influence of the mean discharge in 1940 and in 1941 on the moment characteristics can be observed. As this phenomenon can also be observed in other discharge series of the Labe catchment rivers and other rivers, the corrections of the statistical characteristics in shorter series were determined analogously to the experience from the 110-year series on the Labe in Děčín (Svoboda, 1964). It should not be forgotten, however, that in spite of several common traits the respective hydrological series also have specific characteristics (e.g., water-yield of 1941 need not be so expressive). Therefore, points of the "empirical" curve of exceeding mean annual discharges should be plotted in the probability paper and statistical parameters should be estimated by the quantile method (Chap. 3). Smaller random deviations of statistical characteristics can be expected that when using the method of moments. In the probability paper in Fig.

5.28 there are theoretical curves of exceedance of mean annual discharges for the period 1891–1970 and for 1931–1970. In both cases the statistical characteristics were determined by the method of quantiles. The noticeable difference in the two curves shows that the random deviations can occur throughout the whole curves of exceedance, which means that the variability of statistical characteristics is not only the result of random occurrence of extreme values of mean discharges.

Much less attention has been paid to the influence of the observation period on the size of the seasonal component of the storage capacity. Votruba and Broža (1974) observed differences in the probability curves at the beginning and the end of the seasonal component in a 40-year series (1931–1970) and 80-year series (1891–1970) on the Berounka – Křivoklát which were (O_p = 25 m^3 s^{-1}) (Fig. 5.29).

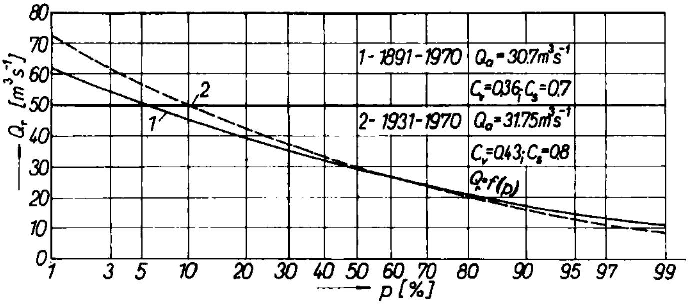

Fig. 5.28 Difference in the theoretical exceedance curves of mean annual discharges (the river Berounka – Křivoklát) with different input discharge observation periods (estimation of statistical characteristics by means of the quantile method)

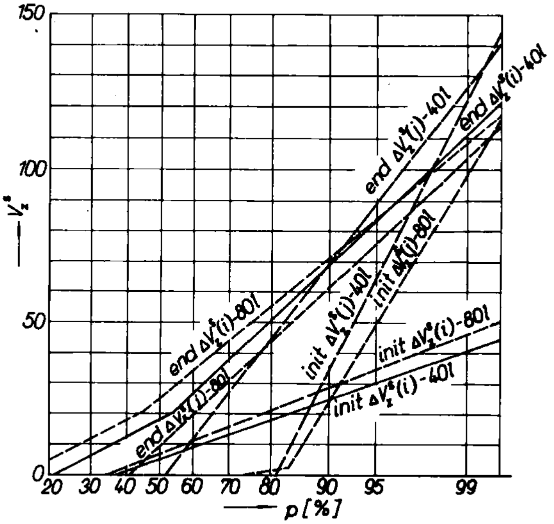

Fig. 5.29 Differences in the probability curves of the respective parts of the seasonal component with various durations of the input discharge series (site Berounka river–Křivoklát)

If we design a reservoir with over-year release control, we must be aware of possible random deviations in the results of the calculations. Even when applying the best available methods to determine the seasonal component, we reach a result which must be considered as an estimation with an error, which on the basis of a 30-year series (e.g., 1931–1960) or a 40-year series (1931–1970) can be $\pm 20\%$, $\pm 30\%$ or even more. Besides this error resulting purely from a limited length of discharge series, there can be others, caused, for example, by derived hydrological data (i.e. estimated, based on the measured data), etc.

The influence of the initial real discharge series can also be observed when synthetic series are used for water management calculations. The observed series helps to estimate the statistical characteristics and correlation bonds, which are the input data of linear regression model of discharge series, and determined the discharge fluctuations within the respective years when using the fragment method, etc.

When modelling the sequence of mean monthly discharges, a large number of numerical constants must be determined from the real series: $3 \cdot 12$ basic statistical characteristics, $12 \cdot 12$ elements of the correlation matrix or further relationships when using the Markov chain of a higher order.

Statistical samples of mean monthly discharges $Q_{m,i}$ have, as compared to mean annual discharges, a much greater variability (e.g., on the site at Křivoklát on the river Berounka the values of C_{vm} for the period 1931–1970 are in the interval $\langle 0.620; 0.964 \rangle$) and also show much greater skewness. The estimation of statistical characteristics $Q_{ma,i}$, $C_{vm,i}$ and $C_{sm,i}$ (i = November, December, ..., October) is very sensitive to any extreme discharges $Q_{m,i}$ in the initial real series; their influence is also unfavourably reflected in the elements of the correlation matrix. This logically leads to the demand that the longest possible discharge series be used as a basis for the construction of a regression model. The variability of the values of $C_{vm,i}$ in relation to the length of the discharge series that was used (80 ÷ 30 years) is shown in Table 5.2. A graphical-numerical analysis of the respective samples of mean monthly discharges can be found very useful.

Table 5.2 Variation coefficient of mean discharges in corresponding months with various lengths of discharge series (the river Berounka–Křivoklát)

Period	XI	XII	I	II	III	IV	V	VI	VII	VIII	IX	X
1891–1970	0.70	0.70	0.71	0.65	0.64	0.68	0.58	0.91	0.95	0.75	0.70	0.62
1901–1970	0.71	0.70	0.71	0.67	0.66	0.66	0.56	0.95	0.97	0.74	0.68	0.60
1911–1970	0.72	0.73	0.74	0.69	0.68	0.68	0.59	0.96	1.00	0.76	0.69	0.63
1921–1970	0.75	0.77	0.60	0.74	0.72	0.68	0.60	0.97	1.03	0.77	0.71	0.60
1931–1970	0.79	0.78	0.64	0.73	0.73	0.72	0.63	0.96	0.96	0.77	0.73	0.62
1941–1970	0.83	0.71	0.69	0.77	0.67	0.74	0.71	1.06	1.05	0.84	0.59	0.59

Other sources of inaccuracies can be the numerical stability of the program, the transformation used, very small or randomly high elements in the correlation matrix, etc. It was found, for example, that synthetic series of mean monthly discharges do not automatically keep the statistical rates of mean annual discharges of the initial real series (Broža, 1973). This simple test of a modelled series is always useful. As a rule it will be necessary to construct several random series and to choose from them those which proved the most suitable in the testing.

The length of a random series is important (for calculations corresponding to a theoretically infinite series. Votruba *et al.* (1971) studied the deviations in the values of the relative yield α and the relative sizes of the storage volume β_z with various lengths of random series (50, 100, 200, 500 years) from the values obtained from a 200-years series. A graphical illustration of the marginal deviations of storage-yield curves

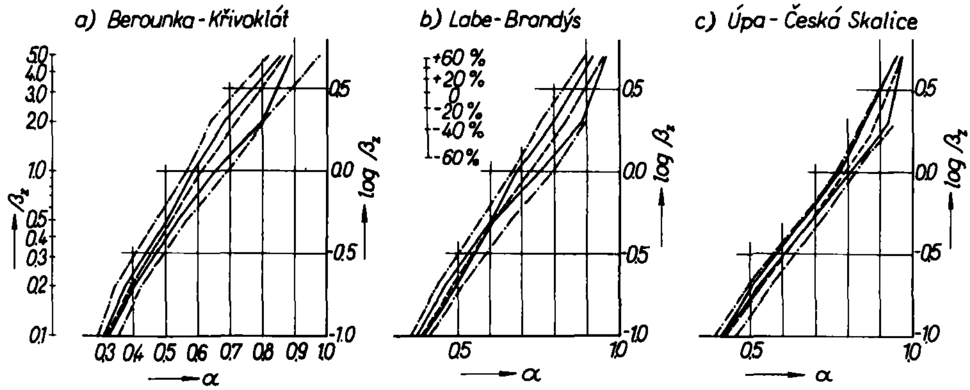

Fig. 5.30 Limit deviations of relation $\beta_z = f(\alpha)$ for $P_o = 99\%$ determined from 500-year and 200-year synthetic series of mean monthly discharges

$\beta_z = f(\alpha)$ with a reliability $P_o = 99\%$ for 200-year and 500-year series can be found in Fig. 5.30. If we do not study the deviations when $\alpha > 0.9$, it is obvious that the relative deviations practically do not change with a change of α or β_z.

Extreme relative deviations $\Delta\alpha/\alpha$ and $\Delta\beta_z/\beta_z$ (in % of values from a 2000-year series) can be found in Table 5.3 and 5.4. For an accuracy of release (withdrawal) of ±5% it will suffice if the random series is 500 years long, or for greater reliability 1000 years long. However, this means that the size of the storage capacity will only be ±25% to ±35% accurate (Table 5.4). If we allow a certain relative error in demands on the storage function of a reservoir (required withdrawal) we can accept a five- to seven-times larger relative error in the size of the storage capacity.

Table 5.3 Extreme deviations of relative yield from the value α_{2000} with various lengths of synthetic series (Berounka r.–Křivoklát, Labe r.–Brandýs, Úpa r.–Č. Skalice)

P_o [%]	Extreme deviation $\Delta\alpha$ (in % of value α_{2000})—length of series								
	100 years			200 years			500 years		
	K	B	S	K	B	S	K	B	S
90	−9	−12	−7	−7	−5	−4	−3	−3	−2
	12	7	8	6	3	3	3	2	1
95	−12	−12	−10	−12	−8	−6	−5	−4	−3
	13	10	8	8	5	2	4	3	2
97	−13	−15	−14	−12	−11	−7	−4	−5	−3
	14	11	7	10	6	5	4	3	4
99				−11	−10	−8	−5	−6	−5
				14	10	9	11	6	4

K Křivoklát–Berounka river B Brandýs–Labe river S Česká Skalice–Úpa river

Table 5.4 Extreme deviations of the relative active storage capacity from the value $\beta_{z,2000}$ with various length of synthetic series (Berounka r.–Křivoklát, Labe r.–Brandýs, Úpa r.–Č. Skalice)

P_o [%]	Extreme deviation $\Delta\beta_z$ (in % of value $\beta_{z,2000}$)—length of series								
	100 years			200 years			500 years		
	K	B	S	K	B	S	K	B	S
90	−35	−25	−33	−29	−15	−13	−22	−10	−7
	45	50	47	38	30	19	12	55	12
95	−52	−31	−30	−35	−19	−12	−16	−10	−8
	65	53	48	70	80	40	20	31	18
97	−56	−38	−31	−40	−22	−24	−22	−18	−21
	58	65	52	73	81	41	24	43	26
99				−45	−37	−31	−39	−34	−24
				66	66	45	26	32	40

For real accuracy of the theoretical size of the storage capacity, there is no need to apply a detailed balancing method to calculate the water losses from a reservoir by evaporation (Chap. 2). In this case we can use more general calculations, such as two time intervals in each year (period of emptying and period of partial filling, or winter and summer period) in which besides the evaporation depth the water pool level of a reservoir must also be considered.

A simpler estimate with the help of the equation

$$Z_v = \bar{H}_E \bar{F} t \tag{5.36}$$

can also be used; here Z_v is the water volume lost by evaporation from the water level during a certain "design" period of reservoir empying, $\bar{H}_E$ – mean daily evaporation depth during the emptying period [m day^{-1}], $\bar{F}$ – estimated mean water

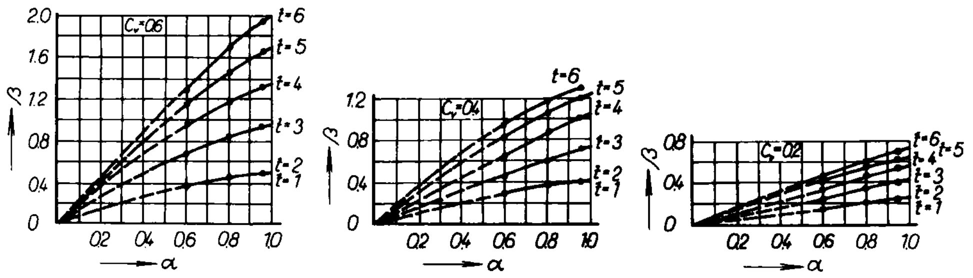

Fig. 5.31 Probable duration of emptying period with over-year release control

level during the emptying period [m^2] (from the characteristics of a reservoir), t – length of "design" period of reservoir emptying.

However, it is always necessary to know the *duration of the reservoir emptying* in the period, which from the point of view of the characteristics of release control can be considered as the design duration. Calculations based on analytical or matrix methods determine the design duration of the "over-year" emptying, either directly during the calculations or by estimation with the help of Kritsky and Menkel's (1952) graphs (Fig. 5.31). The direct method in random discharge series makes it possible to select one or several over-year low-flow periods, according to which the time of emptying typical for the given parameters of release control (α, β_z, P) can be reliably estimated. It might also suffice to use only the most critical period of the real discharge series.

In Czechoslovakia the supplementary volume needed to cover evaporation losses in over-year release control can come to more than 20% of the theoretical size V_z (with relatively flat reservoirs), however, usually it is less (no more than 15%).

When the water losses from a reservoir are an important part of the total balance of a reservoir regime, e.g., with flat reservoirs in dry, warm regions, these losses should be considered as part of the release (withdrawal) from a reservoir, which makes the calculations more complicated (the approximation method must be used).

The error in the theoretical size of the storage capacity arising from the application of series of mean monthly discharges instead of more detailed, e.g., mean daily discharges, is not very great in over-year release control. It can be presumed (see Chap. 6) that the increase in the ordinates of the probability curves of the respective parts of the seasonal component, when using mean daily discharges, will not be greater than 10% as compared with the results gained from mean monthly discharges (the over-year component does not change). As the seasonal component is only a certain part of the theoretical size of the storage capacity (its share can be estimated), the total error will only be a small percentage.

After the theoretical size of the storage capacity has been determined quantitatively (with a relatively large probable error) and after adding volumes for water losses, the resultant size of the storage capacity A_z must be determined. Here not only the water-management aspects, but other aspects connected with reservoir design must be considered, such as the rate of exploitation of a given locality from the morphological, hydrological, etc., points of view, the importance of a reservoir in the complex of water resources in a given region, real conditions of the future reservoir operations, expected development of water demand in the future, development of the complete water-management system in a region and the construction of a reservoir in this system as well as the construction of further water resources. Such an analysis gives a comprehensive view of the design of a reservoir based so far on isolated calculations, as well as a qualitative evaluation of any reserves in the whole system of water resources; this facilitates decisions on the measure of reliability of the design.

The simulation method for the over-year release control using real discharge series cannot give satisfactory results. Even though in recent years great attention has been paid to discharge series of the period 1931 to 1960, and in several important discharge-gauging sites comparative calculations of the classical method (in real series) and statistical methods were carried out, the conclusions cannot be generalized.

In real discharge series, for example, the storage function of a reservoir should be compared with the characteristics of release control determined by probability methods. It can be used for a quick comparison of alternative designs in preliminary studies, for the solution of problems connected with the reservoir operations, etc.

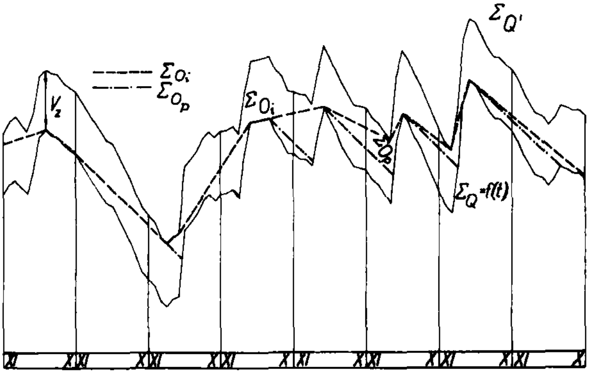

Fig. 5.32 Ideal diagram of over-year release control

The literature (e.g., Morozov, 1954) describes over-year release control in which the storage volume is used not only for its basic function of augmenting the discharge to the required withdrawal O_p, but also for the best possible discharge regulation during high-flow periods. This method is illustrated in Fig. 5.32, in which the mass curve of inflow to a reservoir, $\sum_Q$, is drawn for several years and at a distance equal to the size of the storage capacity V_z an equidistant $\sum_{Q'}$ is drawn. If we draw the mass curve of release from a reservoir $\sum_{O_i}$ as a "tight thread" between the curve $\sum_Q$ and $\sum_{Q'}$, we obtain a very balanced release. As for such a release control, discharges have to be known for several years in advance, it cannot be applied to real reservoir operations. From the method of constructing the mass curve release $\sum_{O_i}$, the term "tight-thread-method" has been derived.