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A nonlinear stability analysis of the coupled equatorial ocean-atmosphere system

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Abstract
A simple model of the coupled ocean atmosphere is considered to investigate the rôle played by nonlinearities in the evolution of large scale instabilities related to ENSO. In this paper we focus on the nonlinear evolution of unstable waves. The initial disturbances develop to finite amplitude on a long space- and time scale and their evolution is governed by amplitude equations of so called complex Ginzburg-Landau type. The coherent structures which emerge show how important the associated space- and time scales are in equatorial dynamics.

1. Introduction
This research is part of the NOP project 853110, entitled: "Non-linear dynamics of the coupled equatorial ocean-atmosphere system". In particular we are looking for the physical mechanisms that determine the behaviour of the tropical coupled system on interannual time scales of 2-9 years (El Niño/Southern Oscillation). Focus is on persistent oscillatory structures in the coupled equatorial Pacific ocean-atmosphere system, developing through non-linear interaction of unstable coupled modes.
As a first approach we study these structures in a non-linear model of the equatorial ocean atmosphere system. Boyd (1980-1987) has already made a rigorous study of the non-linear development of equatorial waves in an uncoupled homogeneous ocean model. On the other hand Hirst (1986) thoroughly investigated the linear stability of the coupled model. The first study shows the presence due to nonlinearities of coherent structures, such as solitons, the second that coupling introduces instabilities which grow in time. Both phenomena have been related to (the onset of) ENSO.
In this study we want to explore the effect of nonlinearities on the initially unstable waves in a weakly nonlinear analysis. Their resulting finite amplitude behaviour is then described by a Ginzburg-Landau equation.

2. Model

To study the weakly nonlinear properties of the equatorial pacific in a qualitative way we use an intermediate coupled ocean-atmosphere model on an equatorial β-plane. This model has a reduced gravity 1.5 - layer nonlinear ocean, describing the temporal and spatial evolution of the horizontal velocities and thermocline depth. The atmosphere is a linear Gill model with horizontal momentum equations and an equation for the evolution of geopotential height. Atmospheric horizontal motions drive oceanic flow, the coupling is completed by a thermodynamic equation describing the evolution of the sea surface temperature (SST), which is optionally nonlinear. The main features of this last equation are advection of SST through zonal oceanic motion and changes in SST trough local thermal processes such as upwelling. The model has a simple climatological background state. Both the oceanic and the atmospheric components have zonally periodic boundary conditions and are meridionally unbounded.

3.Linear stability

Due to the positive feedbackloop induced by coupling in our model ultra long oceanic waves of low frequency are the first to destabilize. We start our linear stability analysis with a simplified thermodynamics equation, considering only local thermal processes associated with upwelling (fast SST-limit). As the coupling is increased the oceanic Kelvin wave becomes unstable, other waves are damped. In fig. 1 the neutral curve, which separates regions of stable (damped) solutions from exponentially growing ones, for this situation is presented. At criticality, at the nose of the neutral curve ($k=k^c=.058$, coupling $m=m^c=.1065$) the wave has a wavelength of 26.000 km, approximately twice the basin width of the equatorial pacific. This unstable coupled Kelvin wave is now dispersive and moving eastward with phase velocity about 66% reduced in comparison to no coupling. In figure 2 eigenfunctions of the Kelvin at critical conditions are shown.
In the case of purely zonal advection of SST, oceanic Rossby waves of wavelengths of about 32.000 km are destabilized, other waves are damped. When the full linear SST equation is considered the dominantly unstable mode is related to the uncoupled mode of this SST equation. Model parameters such as atmospheric damping, and the ocean- atmosphere Kelvin wave speed ratio or a steady state atmosphere affect the solutions only in a quantitative way (cf. Hirst).

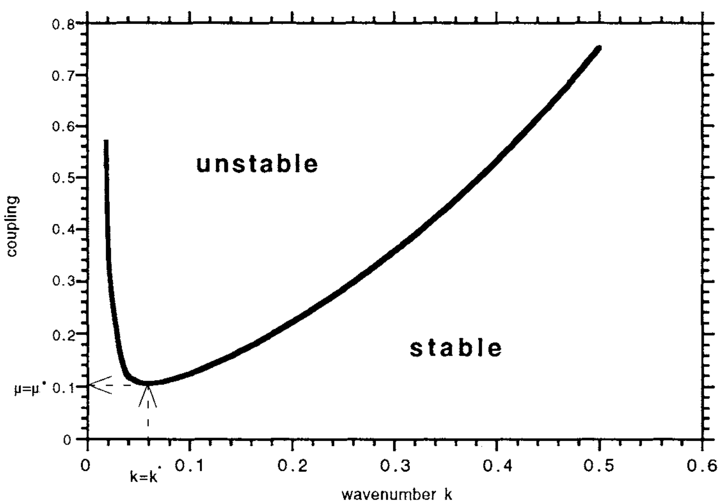

fig. 1 Neutral Curve of the Coupled Kelvin wave

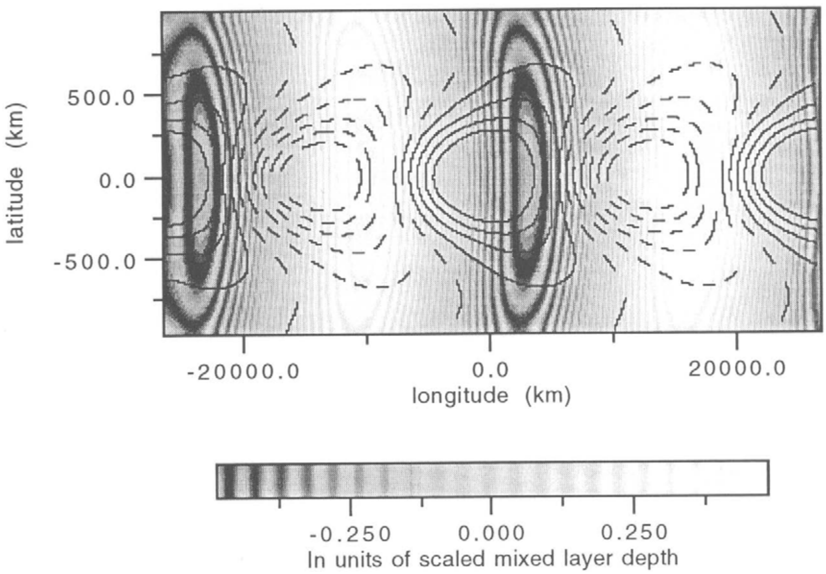

Fig. 2 The eigenfunctions of the Kelvin wave at critical conditions. The coloured background represents the position of pressure highs (light) and lows (dark). The contours indicate mixed layer depth, solid lines are associated with downwelling and positive SST perturbation. Note the positioning of pressure lows just east of positive SST perturbations.

4. Non linear stability

In the previous section results of the linear stability analysis have been sketched, indicating that for different types of SST dynamics a spectrum of wavelike perturbations with exponentially growing amplitudes will develop. This description is valid in the initial growth stage, with very small amplitudes. To investigate the behaviour of these modes on long time- and space scales, the nonlinear terms in the model have to be taken into account. This is done using standard nonlinear analysis techniques.
At criticality the method of multiple scales is used to derive a modulation equation which incorporates the nonlinear self-interaction of the neutral modes. The amplitude A of the initially unstable waves is then governed by the complex Ginzburg-Landau equation

$$\frac{\partial A}{\partial T} = \beta_1 \frac{\partial^2 A}{\partial X^2} + \beta_2 A - \beta_3 A|A|^2 \tag{4.1}$$

Here X and T have been defined on long time and space scales, the complex coefficients β_j depend on all model parameters. To obtain finite amplitude solutions of (4.1) the real parts of β_2 and β_3 have to be positive. In fig.3 the coefficients have been listed for each of the unstable waves. Though the coefficients of the linear terms are accurate, the nonlinear coefficients still need to be checked thoroughly. This and, subsequently, the verification of solutions obtained from equation (4.1) can be done by using a numerical model. At this moment investigation of the amplitude equation for the Kelvin wave is in progress showing periodic solutions, while finite dimensional models of this equation reveal that a-periodic and chaotic solutions are also possible.

unstable mode	crit. wavenumber	β_1	β_2	β_3
Kelvin	.058	4.833+2.065i	0.245+0.0574i	0.00499-0.0331i
Rossby	.048	0.998-1.021i	0.00133+0.00212i	-9.929+15.075i
SST	.042	9.685-2.5278i	0.0516-0.00474i	0.029-0.0046i

Fig. 3 The coefficients β_j for the unstable waves. Note that the Rossby wave has $Re(\beta_3) < 0$, this implies an unbounded amplitude in finite time.

5. Conclusions

We have performed a linear stability analysis of an intermediate coupled equatorial ocean atmosphere model and re-established the most important results found by Hirst (1986). These include the unstable coupled Kelvin, Rossby and SST-modes. Consequently the weakly non-linear behaviour of the model is investigated resulting in the derivation of an amplitude equation of complex Ginzburg-Landau type. This equation governs the evolution of the amplitude of each of the most unstable waves on long time and space scales and inhibits rich dynamical behaviour. The first results show that the amplitude of a Kelvin wave that becomes unstable through local thermal processes grows to a finite value; the solution is then periodic in time and space. Also there are regions in wavenumberspace in which aperiodic and chaotic behaviour occurs. Similar results can be expected for the modulations of the other unstable waves. However, the focus of our study will be to extend this model to include more realistic features of the coupled equatorial pacific, such as zonal boundaries and a spatially varying climatology. Furthermore the oceanic part will be improved by adding more complete thermodynamics and mixed layers. The already established results and developed techniques will be useful in understanding the more complicated models.

References

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