Chapter 4

RECREATIONAL VALUES, PARETO OPTIMALITY AND TIMBER SUPPLY

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1 INTRODUCTION

Recently, there has been an increasing awareness of the fact that a forest fulfills many functions besides producing commercial timber. Very often, one would expect forest land to provide not only different, but also conflicting services. The classical example is a wilderness area, which can be left unspoiled and used for various recreational purposes or it can be commercially exploited by harvesting the trees. Similarly, the attractiveness of the environment may depend on the age (distribution) of trees, implying that the "socially" optimal rotation period need not coincide with the Faustmann rotation period.

In a recent paper, Johansson and Löfgren (1988), money measures were introduced of the total value of a forest supplying many different and possibly conflicting services. Such money measures are, in principle, possible to estimate by using, for example, questionnaire techniques.

In this context it was pointed out that the externality in consumption created for agents with preferences regarding the environmental services provided by the forest, but no direct control over the management of the trees, must be dealt with through "special methods" in a market economy. This is because the optimal provision of environmental services normally conflicts with commercial management practices. The point is that present value maximizing forest owners must be induced to manage the forest in a manner that is consistent with Pareto optimality.

As we will demonstrate, this goal can, under certain conditions, be accomplished by a system of individually based shadow prices (pseudoequilibrium prices) which creates relative "timber prices" that are consistent with a welfare optimum. This was first shown in a more general environmental context by Mäler (1974)[1]. It means that we can decentralize the social optimization problem by adding an environmental component to the ordinary present value problem. A special point being that this component is linear in the stand variables. In other words, the problem mentioned by Hartman (1976) that "For many plots of forest land which could reasonably be taken as unit for making cutting decisions, what happens on one plot will

[1] Similar results are found in Foley (1970) and Milleron (1972). See also Mäler (1985).

clearly affect the value of a standing forest on other units", is automatically solved by the information provided by the pseudoequilibrium prices and decentralized present value maximization. Bowes' and Krutilla's (1985) concern that "it would seem most unrealistic to assume that a single stand could be considered independently of the condition of adjacent stands" is understandable; but this is also taken care of by the pseudoequilibrium prices. Since the equilibrium is also a Pareto optimum, there cannot exist any arbitrage possibilities which, if present, would improve the situation for at least one individual, the arbitrator[2]. In other words, all interactions are already reflected in the prices.

The practical relevance of the management problem cannot be exaggerated. For example, the Forest and Rangeland Renewable Resource Planning Act of 1974, as amended by the National Forest Management Act of 1976, "imposes" on the US–Forest Service a legislation mandate to conduct forest management under both commercial and broader environmental considerations. The analytical results derived in this paper pertain to a FORPLAN[3] solution under perfect foresight and known shadow prices of environmental services. This is indeed an ideal situation, but this does not obliterate the need for theoretical information on the properties of the optimal harvesting program. Moreover, recent progress in the economic theory and measurement of environmental benefits has been considerable (for a survey, see Johansson (1987)). Thus a meaningful practical application of the approach may not be too far fetched, since even incomplete information on environmental benefits combined with qualitative theoretical information can be used to improve welfare. For example, the (qualitative!) information that the value of the environmental services provided by a forest stand increases with the age of the stand, combined with the qualitative theoretical information on the direction in which this changes the optimal rotation period[4] can be used to conclude that a rotation period greater than the commercial rotation period is welfare improving.

The remaining part of this paper is structured as follows: In section 2 we show, within a very simple text-book model of a small open economy with two agents, one a forest owner deriving (indirect) utility only from the commercial services of the forest, and the other a worker deriving utility also from the environmental services of the forest, how a welfare optimizing system of shadow prices (or prices for environmental services) can be designed[5]. The idea is to give the reader an intuitive feeling for how things would work in a more general

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2 Compare the Value Additivity Theorem in financial economics, which states that: If no arbitrage possibilities exist, then the price of a security whose pay-offs are a linear combination of other assets must be given by the same linear combination of the prices of the other assets. See e.g., Varian (1987).

3 The US–Forest Service planning model. See also Bowes and Krutilla (1985).

4 See e.g. Bowes and Krutilla (1985) or Hite et al (1987).

5 A considerably more comprehensive analysis can be found in Mäler (1974).

setting. In Section 3 we turn to the microeconomics of the management problem. We show how the augmented present value maximizing problem, containing the demand determined shadow prices of forest land in different age classes (from Section 2), can be solved. In particular, we derive the properties of the present value function and show when and why an efficiency criterion on the intertemporal supply of timber may be violated.

Section 4 contains a discussion of how the pseudoequilibrium prices can be approximated without making system wide calculations, and how forest management can be induced to converge towards the neighborhood of a socially optimal program. One idea is to use a steady state estimate of the pseudoequilibrium prices in "the normal forest"[6] to generate a cutting policy that will asymptotically approach the social optimum.

2 THE ENVIRONMENTAL VALUE OF TREES AS AN EXTERNALITY IN CONSUMPTION

In order to highlight the economic problems created in a situation where agents derive utility from the environmental services produced by the forest, as well as from the commercial values obtained through the timber harvest, we will introduce a very simple general equilibrium model. It contains two ordinary goods, a consumer good, x, and timber, c. Both goods can be sold in an international market at prices p and P, respectively. Timber is produced through the input of environmental services e and labor l. The price of labor, w, is determined in a competitive labor market, while environmental services are unpriced[7].

There are two representative agents in the economy, a forest owner, and a worker. The forest owner produces timber and consumes food and derives no satisfaction from the environmental services of the forest. His optimization problem can be formulated in the following manner:

$$v\left[x_1(P,p,w)\right] = \underset{x_1, l^d, e}{\text{Max}} \{u(x_1) \mid Pc(e, l^d) - px_1 - wl^d = 0\} \tag{1}$$

where $Pc(e, l^d) - px_1 - w\, l^d = 0$ is his budget constraint. $Pc(e, l^d)$ can be interpreted as the present value of all future timber rotations, while px_1 and wl^d are the cost of food and the labor input, respectively. This formulation is implied by a perfect capital market. Finally, $v\left[x_1(P, p, w)\right]$ is the so–called indirect utility function with properties:

6 The normal or "syncronized" forest is defined as a forest with stands of equal area and exactly one stand in each age class up to the optimal rotation period. Such a steady state means that the same area and volume – assuming a constant biotechnology – will be cut every year.

7 An alternative way to set up the problem would be to assume that the supply of environmental services is a function of the labor input.

$$v_p = -\lambda x_1(P, p, w)$$
$$v_P = \lambda c(P, p, w) \qquad (2)$$
$$v_w = -\lambda l^d(P, p, w)$$

where $\lambda(P, p, w)$ can be interpreted as the marginal utility of income. In other words, the demand for food and labor, and the supply of timber can be derived (except for sign) by differentiating the indirect utility function with respect to the respective price and dividing by the marginal utility of income.

The supply of timber and environmental services as well as the demand for labor are determined from the maximization of the present value of the income from forestry. The first order conditions are:

$$\left.\begin{aligned} P\frac{\partial c(\cdot)}{\partial l^d} - w &= 0 \\ P\frac{\partial c(\cdot)}{\partial e} &= 0 \end{aligned}\right\} \Rightarrow \begin{aligned} l^d &= l^d(P, w) \\ e &= e(P, w) \end{aligned} \qquad (3)$$

which mean that the intensity in forestry depends only on the price of timber and the wage rate and not on the price of consumer goods. This is a consequence of the perfect capital market assumption, and the resulting separation property is often referred to as the "Fisherian separation theorem"[8].

The optimization problem of the worker is formulated as:

$$V(p, w, e) = \underset{x_2, l^s}{\text{Max}} \; \{U(x_2, l^s, e) \mid wl^s - px_2 = 0\} \qquad (4)$$

where wl^s is labor income from supplying l^s hours of work. The derivatives of the indirect utility functions are:

$$V_p = -\mu x_2(e, p, w)$$
$$V_w = \mu l^s(e, p, w) \qquad (5)$$
$$V_e = \frac{\partial U(e, p, w)}{\partial e}$$

where $\mu(e, p, w)$ is the worker's marginal utility of income. The prices of timber (P) and food (p) are determined in the international market, while the wage rate is determined by the market clearing condition in the labor market.

[8] For its origin see Fisher (1930).

$$l^d(P, w) = l^s(e, p, w) \tag{6}$$

Note also that prices appear in the marginal utility of environmental services, since l^s and x_2 in optimum depend on prices.

The first six equations now determine the market equilibrium. However, due to the externality in consumption created by the fact that the environmental services are imposed on the worker, the market equilibrium is not necessarily a Pareto optimum. In order to see this, assume that $\frac{\partial U}{\partial e} \neq 0$ in the market equilibrium, say that it is positive; meaning that a (small) increase in the supplies of environmental services will improve the worker's situation. Moreover, since $P\frac{\partial c}{\partial e} = 0$ in equilibrium, a small increase in the value of e will leave the income and utility of the forest owner unaffected. Hence, welfare can be improved according to the Pareto criterion. This potential improvement is not materialized in the market solution, since there is no formal way for the worker to communicate a willingness to buy additional environmental services ($\frac{\partial U}{\partial e} > 0$)[9].

How would we achieve a Pareto optimum? The firm's marginal willingness to pay for environmental services is given by:

$$\beta\Big[e, P, p, w(e, P, p)\Big] = P\frac{\partial c}{\partial e}\Big[e, l^d\big[P, w(e, P, p)\big]\Big] \tag{7}$$

The argument e in the wage function is a consequence of treating e as a parameter in the optimization problems, which in turn implies that the equilibrium wage determined in (6) will be a function of e.

The worker's marginal willingness to pay for the environmental services in monetary terms is given by:

$$\alpha(e, P, p) = \frac{\partial U(e, p, P)}{\partial e}\,\mu\Big[e, p, w(e, P, p)\Big]^{-1} \tag{8}$$

The total marginal willingness to pay is hence given by the sum of the expressions in equations (7) and (8), respectively, which if equalized to zero enables us to solve for the optimal provision of environmental services. (Note that P and p are exogenously given.)

$$\beta(e^*, P, p) + \alpha(e^*, P, p) = 0 \tag{9}$$

9 Note that the public good property of environmental services means that in an economy with more than one agent caring about the environment the "marginal willingness to pay" of different agents has to be added. See below.

This "Lindahl equilibrium"[10] volume of environmental services, e^*, means that the worker has gained at the expense of the forest owner. The former would, however, be willing to pay $\alpha(e^*)$ dollars for the last unit of the environmental good and this amount would exactly compensate the latter's loss from producing it. In other words, a policy that promised the forest owner $\alpha(e^*)$ dollars per unit of the environmental good, would induce him to produce[11] the optimal amount e^*. The lump–sum of money $\alpha(e^*)\cdot e^*$ is also the minimum amount of money that would accomplish this, and that the worker would be willing to pay, since[12] $\alpha(e) > \alpha(e^*)$, for $e < e^*$. In other words, given $\alpha(e^*)$ the consumer demands e^*.

If we introduce many consumers of the environmental services, the optimal allocation problem can be solved by adding the respective marginal willingness to pay functions, $\alpha_i(\cdot)$, $i = 1, ..., n$, and using the sum in (9) to solve for e^*. The firm's shadow price becomes $\alpha = \sum_{i=1}^{n} \alpha_i(e^*)$, and the costs to consumers are allocated according to the principle that each agent pays (or receives if $\alpha_i < 0$) $\alpha_i(e^*)e^*$.

The above is a sketchy and simplified treatment of a general allocation problem involving public goods. The existence of a Lindahl equilibrium, which is crucial for the analyses to follow, was first dealt with thoroughly in Milleron (1972). Mäler (1974) solved a corresponding problem[13] with explicit reference to environmental services. In this general setting α and e would be vectors, and environmental services can, of course, be dated, so we can apply the model to an economy where trees are produced, and where forest stands, as such, produce environmental services. A Lindahl equilibrium means that all markets are cleared and all consumers demand the same amount of environmental services. It is also possible to show, in the spirit of Arrow-Debreu, that this equilibrium is a Pareto optimum and that every Pareto optimum can be represented as an equilibrium.

Loosely speaking, the necessary and sufficient condition for the validity of this general theorem is that each producer is limited to a convex production set, and that each consumer is guided by a convex preference ordering. Under these assumptions the aggregate production and consumption sets can be separated by a price-hyperplane.

Some strong arguments have been given against assuming that the aggregate production, set under the existence of detrimental externalities, is convex. Nonconvexities would prevent the establishment of optimal prices in voluntary exchange. However, it would not prevent

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[10] See Lindahl (1919), Foley (1970), and Milleron (1972).

[11] The first order condition for present value maximum

$P\, y_e(P, p, e^*) + \alpha(e^*) = 0$

would be fulfilled.

[12] Certain regularity conditions are implicitly assumed to hold.

[13] An "unimportant" difference being that Mäler introduces an environmental agency that supplies the environmental services to the consumers. The existence of Lindahl equilibrium is also discussed by Hart and Kuhn (1975).

the efficiency properties of taxes representing the social marginal benefits of environmental services[14]. Whether nonconvexities are also crucial for the production of the recreational services of the forest must be the topic of another paper. Here we will proceed as if an intertemporal equilibrium exists.

3 THE MICROECONOMICS OF FORESTRY IN THE PRESENCE OF ENVIRONMENTAL SERVICES

Having established the existence of a price vector, including prices for environmental services, which clears all markets and creates Pareto optimality, we will redo the Hartman (1976) rotation analysis in a more general and appropriate setting. The model we introduce is a variation of the Berck-Johansson – Löfgren[15] linear and nonlinear forest management model. This has been augmented with an extra term in the goal function which contains a valuation of the age distribution of the forest[16]. The assumption being that an age class has given environmental attributes which can be priced.

Time has now come to introduce the technology of the forestry firm. Let

x_{ti}	= the number of acres of land occupied at the end of period t by trees in age class i
x_t	$= [x_{t0}, ..., x_{tn}]$
c_{ti}	= the number of acres of land with trees of age class i cut in period t
c_t	$= [c_{t1}, ..., c_{tn}]$
M	= the total amount of forest land
g_i	= the amount in m^3 that can be harvested from one acre of land with trees in age class i
g	$= [g_1, ..., g_n]$
r_{ti}	= the price of one m^3 of timber from trees in age class i at time t
q_{ti}	= the price of timber from trees on one acre with trees in age class i at time t, i.e., $q_{ti} = r_{ti}\, g_i$.
q_t	$= [q_{t1}, ..., q_{tn}]$
r	= the interest rate in the perfect capital market
$K_t(c_t)$	= the cost of cutting $[c_{t1}, ..., c_{tn}]$ in period t
$E_t(x_t)$	= the net social benefits in period t from having an age distribution of the forest given by x_t at the end of period t.

[14] This is shown by Starret (1972).

[15] See Berck (1976) and Johansson and Löfgren (1985). To give the model a name is a little pretentious, since at least the linear version is a standard LP–model of a multiple stand forest.

[16] The model is also in its non–linear version a generalization of Bowes–Krutilla (1985), who take the problem considerably further than Hartman (1976) and Strang (1983). Another difference compared with earlier models is that we interpret prices as corrected general equilibrium prices that create Pareto optimality, through a well defined market induced mechanism. In other words, multiple use forestry is married to the Mälerian general equilibrium approach.

The basic dynamics of the growth of the forest is given by the following equations:

$$\sum_{i=0}^{n} x_{0i} = M \tag{10}$$

$$x_{t0} - \sum_{i=1}^{n} c_{ti} = 0 \qquad t = 1, \ldots, T \tag{11}$$

$$x_{t-1\ i-1} - c_{ti} - x_{ti} = 0 \qquad \begin{matrix} t = 1, \ldots, T \\ i = 1, \ldots, n-1 \end{matrix} \tag{12}$$

$$x_{t-1\ n-1} + x_{t-1\ n} - c_{tn} - x_{tn} = 0 \qquad t = 1, \ldots, T \tag{13}$$

$$\bar{x}_{0i} - x_{0i} = 0 \qquad i = 0, \ldots, n \tag{14}$$

where we have assumed that a tree that has reached age class n will stay there forever unless it is cut. A feasible cutting policy must also satisfy the inequalities c_{ti}, $x_{ti} \geq 0$ for all t.

The net social benefits in period t can be written:

$$q_t c_t - K_t(c_t) + E_t(x_t) \tag{15}$$

The present value of all the future benefits is then:

$$\sum_{t=1}^{T} [q_t c_t - K(c_t) + E_t(x_t)]\ [1 + r]^{-t} \tag{16}$$

The optimal management of the forest in the face of the environmental considerations is defined as the cutting policy that maximizes the present value.

In order to characterize such an optimal policy, the Kuhn-Tucker conditions will be used. The Lagrangian of the optimization problem is:

$$\begin{aligned} L = & \sum_{t=1}^{T} ([q_t c_t - K_t(c_t) + E_t(x_t)]\ [1 + r]^{-t}) + \sum_{i=0}^{n} (\lambda_{0i}[\bar{x}_{0i} - x_{0i}]) \\ & + \sum_{t=1}^{T} (\lambda_{t0}(\sum_{i=1}^{n} c_{ti} - x_{t0})) + \sum_{t=1}^{T} \sum_{i=1}^{n-1} (\lambda_{ti}[x_{t-1\ i-1} - c_{ti} - x_{ti}]) + \\ & \sum_{t=1}^{T} \lambda_{tn}[x_{t-1\ n-1} + x_{t-1n} - c_{tn} - x_{tn}] \end{aligned} \tag{17}$$

The necessesary conditions for a maximum are:

$$\begin{array}{lll}
\text{(i)} & \dfrac{\partial L}{\partial c_{ti}} = [q_{ti} - \dfrac{\partial K_t}{\partial c_{ti}}]\,[1 + r]^{-t} + \lambda_{t0} - \lambda_{ti} \le 0 & c_{ti}\,\dfrac{\partial L}{\partial c_{ti}} = 0 \\
\text{(ii)} & \dfrac{\partial L}{\partial x_{0i}} = -\lambda_{0i} + \lambda_{0i-1} \le 0 & x_{0i}\,\dfrac{\partial L}{\partial x_{0i}} = 0 \\
\text{(iii)} & \dfrac{\partial L}{\partial x_{ti}} = \dfrac{\partial E_t}{\partial x_{ti}}\,[1 + r]^{-t} + \lambda_{t+1\; i+1} - \lambda_{ti} \le 0 & x_{ti}\,\dfrac{\partial L}{\partial x_{ti}} = 0 \\
\text{(iv)} & \dfrac{\partial L}{\partial x_{tn}} = \dfrac{\partial E_t}{\partial x_{tn}}\,[1 + r]^{-t} + \lambda_{t+1n} - \lambda_{tn} \le 0 & x_{tn}\,\dfrac{\partial L}{\partial x_{tn}} = 0 \\
\text{(v)} & \dfrac{\partial L}{\partial \lambda_t} \ge 0 & \lambda_t\,\dfrac{\partial L}{\partial \lambda_t} = 0 \\
\text{(vi)} & c_t,\, x_t \ge 0 &
\end{array} \tag{18}$$

If the cost functions are convex these conditions will also be sufficient. The Lagrange multipliers $\lambda_t = [\lambda_{t0}, \ldots, \lambda_{tn}]$ (the royalties) can be interpreted as the net present value of standing trees at time t, given that the stands are managed according to the optimal program. Roughly speaking (18 i) tells us that if it is optimal to cut a stand at t ($c_{ti} > 0$), then the net marginal revenue from the last acre plus the value of newly seeded land (λ_{t0}) coincides with the royalty (λ_{ti}).

If some of the trees are saved, $x_{ti} > 0$, then the present value of the marginal recreational value will coincide with the difference between the royalty in period t and the corresponding royalty of the same stand in a later period. Since the recreational value is positive this means that the royalty will fall over time.

It is easily seen that if

$$\alpha_{ti} = \frac{\partial E_t(\cdot)}{\partial x_{ti}} \qquad t = 1, \ldots, T \quad i = 1, \ldots n \tag{19}$$

are considered constant psedoequilibrium prices along the optimal trajectory, then the same optimal management plan of the forest would be obtained if $\sum_i \alpha_{ti} x_{ti}$ is substituted for $E_t(x_t)$ in the objective function.

Assuming for the moment that the psedoequilibrium prices are known and that we are dealing with a linear version of (17), which is obtained if we drop the cost functions, and reinterpret the price vector as consisting of net present values. By manipulating the Kuhn-Tucker conditions (18), which are both necessary and sufficient in the LP–case, it is easy to prove the following three propositions (defining $q_{ti}(1+r)^{-t} = p_{ti}$ and $\alpha^r_{ti} = \alpha_{ti}(1+r)^{-t}$).

<u>Proposition 1</u> The royalties of the combined present value LP–problems can be found by using the algorithm

$$\lambda_{ti} = \max\,[\lambda_{t+1\;i+1} + \alpha^r_{ti}, p_{ti} + \lambda_{t0}]$$

for all t (starting with t=T) and all i < n. For i = n the right hand side should be written Max $[\lambda_{t+1n} + \alpha^r_{tn}, p_{tn} + \lambda_{t0}]$.

The modification that has to be made in comparison with a pure present value program is that the value of saving the trees consists not only of the royalties in the next period, but also the demand price of the environmental services consumed during the period (α^r_{ti}).

From the way in which the solution in Proposition 1 is obtained, part (a) of the following proposition is obviously true, provided that the algorithm in Proposition 1 can be started with scrap values that are independent of x_0.

<u>Proposition 2</u> (a) The royalties λ_{ti} are independent of the initial endowments $(\bar{x}_{00}, ..., \bar{x}_{0n})$.
(b) The royalties are non–decreasing functions of the price vectors $p_1, p_2, ..., p_T$ (which contain the coefficients of the production function $g_1, g_2, ..., g_n$), and the psedoequilibrium price vectors $\alpha^r_1, \alpha^r_2, ..., \alpha^r_T$.

Part (b) of Proposition 2 is intuitively true[17]. If prices increase in the future, the value of a unit of forest land today cannot decrease. On the other hand, only prices in the future matter. Yesterday's prices (timber or prices of environmental services) cannot affect the combined value of a unit of today's forest land. Moreover, a better production function cannot, ceteris paribus, decrease the combined value of forest land.

Intuition also suggests that the stand may be profitably cut if

$$\lambda_{ti} = p_{ti} + \lambda_{t0} \geq \lambda_{t+1\;i+1} + \alpha^r_{ti} \tag{20}$$

i.e. if the value of the cut plus the value of the seed or a unit of forest land is worth not less than the royalty of the same unit of forest land at time t+1 plus the value of the environmental services supplied during period t. A more formal argument, making use of the Lagrangian of the forest management problem and the Kuhn–Tucker condition, reveals that intuition is indeed correct, i.e.:

<u>Proposition 3</u> The cutting (saving) rules of the combined present value LP–problem are the following

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[17] The second part of the claim can be demonstrated by an induction argument.

(a) Timber of age class i is cut at time t if

$$\lambda_{t0} + p_{t+1\ i+1} > \lambda_{t+1\ i+1} + \alpha^{r}_{ti}$$

(b) It may be saved or cut at time t if

$$\lambda_{t0} + p_{ti} = \lambda_{t+1\ i+1} + \alpha^{r}_{ti}$$

(c) Timber is saved if

$$\lambda_{t0} + p_{ti} < \lambda_{t+1\ i+1} + \alpha^{r}_{ti}$$

Since the cutting rules do not depend on the composition of the initial endowment, we have the following Corollary:

Corollary 1 The optimal management of a forest stand in the combined present value problem, does not depend on the size and composition of $\bar{x}_0$.

One can say that Corollary 1 is the theoretical justification for the practice, emerging from Hartman (1976), of treating the combined present value maximization problem stand by stand.

Things are more complicated if we take account of the cost functions. For example, if the cutting technology exhibits increasing or decreasing returns to scale[18], it may be profitable to merge cuttings together, and optimal harvesting decisions can no longer be taken stand by stand. Note, however, similar problems do not enter into the environmental part of the combined present value function. Since everyone consumes environmental services in the same quantities (the public good property), and the environmental prices by assumption are sums of individual Pareto–supporting valuations in the same sense as competitive timber prices, they can be used as ordinary competitive prices. In other words, even if the consumers value forest land as a whole, with age composition mattering for the aggregate value, this is implicit in the environmental prices. Like competitive equilibrium prices, they pick up all the interactions of the economy.

Let

$p = [p_1, p_2, \ldots p_T]$ $\qquad\qquad$ $\alpha = [\alpha^r_1, \alpha^r_2 \ldots \alpha^r_T]$

$c' = [c_1, c_2, \ldots, c_T)$ $\qquad\qquad$ $x' = [x_1, x_2, \ldots, x_T]$

$C = \{c, x \mid c, x \text{ satisfy eqs } (10) - (14)\}$

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18 Economics of scope, a restricted form of sub-additivity, would create a similar deviation from a stand by stand treatment. See Baumol, Panzar and Willig (1982) ch. 4.

Moreover, we introduce a more explicit cutting technology by letting the labour input[19] in period t be a function of the cutting program c_t.

$$\ell_t = f_t(c_t)$$

Finally, we introduce the input vector and the input price vector:

$$\ell = (\ell_1, ..., \ell_T) \qquad\qquad w' = (w_1, ..., w_t)$$

where w_i is the present value of the cost of a unit of labor in period t, and a feasible set F such that:

$$F = \{\ell \mid \ell_t = f_t(c_t), \forall\, t\}$$

We are now ready to define a "combined present value function".

$$\pi(p, w, \alpha) = \underset{c\,,\,\ell,x}{\text{Max}} \;\{pc - w\ell + \alpha x \mid c, \ell, x, \epsilon\; C \cap F\}.$$

The fundamental properties of the combined present value function are summed up in the following proposition.

<u>Proposition 4</u>

(a) The combined present value function is convex in prices $\theta = (p, w, \alpha)$, i.e. $\lambda\pi(\theta') + (1-\lambda)\;\pi(\theta'') \geqq \pi(\theta^\lambda)$, where λ is a scalar $0 < \lambda < 1$, and $\theta^\lambda = \lambda\theta' + (1-\lambda)\;\theta''$.

(b) It is homogeneous of degree one in prices, i.e., $\mu\pi(\theta) = \pi(\mu\theta)$, for $\mu > 0$.

(c) It is subadditive in prices, i.e., $\pi(\theta') + \pi(\theta'') \geq \pi(\theta' + \theta'')$.

Claims (a)–(c) are also properties that are held by the profit function in neoclassical theory, and the proofs are standard[20] and therefore omitted. Convexity means, among other things, that the function is continuous and almost everywhere twice continuously differentiable on the positive orthtant. Homogeneity means that a doubling of all prices doubles the profit, and homogeneity and convexity implies subadditivity, but a direct proof of the subadditivity claim is also easy.

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[19] A more general input vector would not cause any additional problems. Note that the cost function $K_t(\cdot)$ is defined as $\underset{\ell_t}{\text{Min}}\{w_t\ell_t \mid \ell_t = f_t(c_t)\} = w_t f_t(\cdot)$.

[20] See e.g. Johansson and Löfgren (1985).

Convexity also means that the quadratic form of the function is negative semidefinite and homogeneity of degree one means that the derivative of the function is homogeneous of degree zero. The usefulness of these properties will be more evident after we have introduced Proposition 5.

Subadditivity has, so far, been a less useful property than the other two, but Löfgren (1987) uses it together with convexity to provide upper and lower bounds on the change in the value of forest land when land value is evaluated at any two different price vectors.

If we define $y = (c, -\ell, x)$ the combined present value function can be written

$$\pi(\theta) = \underset{y}{\text{Max}} \, [\theta y \mid y \in C \cap F]$$

The following proposition can now be proved.

<u>Proposition 5</u> For an interior solution, $\theta >> 0$, it is true that $D\pi(\theta) = y(\theta)$, when the derivatives exist (where $D\pi(\theta)$ is the gradient of the profit function).

Proof: Define a function

$$h(\theta) = \theta y' - \pi(\theta)$$

where y' maximizes $\pi(\theta)$ for $\theta = \theta'$. This function will, by definition, have a maximum for $\theta = \theta'$, and since it is an interior maximum it holds that

$$\frac{dh}{d\theta}(\theta') = y' - D\pi\,(\theta') = 0$$

Since this is true for all θ' the claim is proved.

Decomposing y we find, $c(\theta)$, the timber supply vector, $x(\theta)$, the environmental supply vector of standing timber and, $-\ell(\theta)$, the labor demand vector. Now since $\pi(\theta)$ is convex, the matrix $D^2\pi(\theta) = D\,y(\theta)$ is a symmetric and positive semidefinite matrix. For the case when there are only two periods and two age classes we have

$$D^2\pi(\theta) = \begin{bmatrix} \frac{\partial c_{11}}{\partial p_{11}} & \frac{\partial c_{11}}{\partial p_{12}} & \frac{\partial c_{11}}{\partial p_{21}} & \frac{\partial c_{11}}{\partial p_{22}} & \frac{\partial c_{11}}{\partial w_1} & \frac{\partial c_{11}}{\partial w_2} & \frac{dc_{11}}{\partial \alpha_{10}} & \cdots & \frac{dc_{11}}{\partial \alpha_{22}} \\ \vdots & & & & & & & & \\ \frac{-\partial \ell_1}{\partial p_{11}} & \frac{-\partial \ell_1}{\partial p_{12}} & \frac{-\partial \ell_1}{\partial p_{21}} & \frac{-\partial \ell_1}{\partial p_{22}} & \frac{-\partial \ell_1}{\partial w_1} & \cdots & \cdots & \cdots & \frac{-\partial \ell_1}{\partial \alpha_{22}} \\ \vdots & & & & & & & & \\ \frac{\partial x_{10}}{\partial p_{11}} & \cdots & \cdots & \frac{\partial x_{10}}{\partial p_{22}} & \frac{\partial x_{10}}{\partial w_1} & & \frac{\partial x_{10}}{\partial \alpha_{10}} & \cdots & \frac{\partial x_{10}}{\partial \alpha_{22}} \\ \vdots & & & & & & & & \\ \frac{\partial x_{22}}{\partial p_{11}} & \cdots & \cdots & \frac{\partial x_{22}}{\partial p_{22}} & \frac{\partial x_{22}}{\partial w_1} & \cdots & \cdots & \cdots & \frac{\partial x_{22}}{\partial \alpha_{22}} \end{bmatrix}$$

From the semidefiniteness of the quadratic form it follows that:

(i) The own supply effect is nonnegative ($\frac{\partial c_{ti}}{\partial p_{ti}}, \frac{\partial x_{ti}}{\partial \alpha_{ti}} \geqq 0$).

(ii) The own demand effect is nonpositive ($\frac{-\partial \ell_i}{\partial w_i} \geqq 0$).

(iii) The crossprice effects are symmetric ($\frac{\partial c_{ij}}{\partial \alpha_{lk}} = \frac{\partial x_{lk}}{\partial p_{ij}}$, or the element ij equals the element ji).

Claims (i) and (ii) tell us that the elements along the main diagonal are positive, meaning, for example, that an increase in the price of an environmental service or timber in a certain age class in a particular period increase the supplies of these services. Claim (iii) means, for example, that an increased demand price for a stand in age class 2 in period 2, (α_{22}), has the same effect on timber supply in age class 1, as an increase in the price of timber in age class 1 in period 1 has on the supply of environmental services in age class 2 in period 2 ($\frac{\partial c_{11}}{\partial \alpha_{22}} = \frac{\partial x_{22}}{\partial p_{11}}$).

Finally, from Proposition 5 and the properties of homogeneous functions it follows that the supply and demand functions are homogeneous of degree zero in prices, i.e., $c(\mu\theta) = c(\theta)$, $x(\mu\theta) = x(\theta)$, and $\ell(\mu\theta) = \ell(\theta)$. In other words, only relative prices matter for the firm's supply and demand decisions.

Let us now introduce the following definition of efficiency.

<u>Definition:</u> A feasible program $(c_t', -\ell_t', x_t')$, $t = 1, ..., T$ <u>dominates</u> another feasible program if $gc_t' \geq gc_t$ and $-\ell_t' \geq -\ell_t$ for all t with at least one strict inequality. A program is production <u>efficient</u> if it is feasible and there is no dominating program. A dominated program is called inefficient.

It is fairly well known, since the classical paper by Malinvaud (1953), that if the planning horizon is infinite the present value maximizing program will not necessarily be efficient. However, as soon as the planning horizon is finite the present value maximizing program is efficient.

It is equally well known that the maximization of combined present value function under both a finite and infinite planning horizon may involve an inefficiency in production. In the infinite time horizon case this has been shown by Strang (1983), and in the finite time horizon case it is obvious from the solution algorithm in the linear case that every time trees are saved in the final period there is an inefficiency in production.

What can easily be shown for the combined present value problem with a finite planning horizon is that the optimal program is efficient if it is present value maximizing. The reverse statement is, however, not in general true.

4 HOW TO FIND THE PSEUDOEQUILIBRIUM PRICES?

The points made so far would only be of pure theoretical interest, if it is necessary to make systemwide calculations in order to find the pseudoequilibrium prices. The problem of finding the correct shadow prices for the environmental services may seem almost insurmountable. We will, in this section, discuss three approaches to a search for these prices, one of which we consider to be a promising possibility.

(i) A particularly simple case would be if the function $E_t(x_t)$ is linear, i.e. the willingness to pay for a hectar of land in a certain age class is independent of the total number of hectars in this age class, and also of the age of trees in surrounding stands. As has been pointed out by among others, Bowes and Krutilla (1985) this is hardly a reasonable case.

(ii) Another possibility would be if other similar forests exist, and the willingness to pay for the recreational services of trees in a certain age class depends on the total number of hectars in the age class. Formally, this can be written as:

$$E_t = E_t(x_t + X_t)$$

where X_t is the aggregate area of all other forests. The marginal willingness to pay for trees of age class i would be

$$\partial E_t(x_t + X_t)/\partial x_{ti}$$

and if x_{ti} is small compared to X_{ti} (the total number of hectares in age class i), it is reasonable to assume that the marginal willingness to pay is approximately independent of x_t. This would correspond to the case when a mountain valley is going to be cut, at the same time as there exist a number of similar valleys. The marginal willingness to pay for preserving trees of a certain age class would, in this case, be determined by the valuations of the age distributions in all other valleys.

Thus, when there exist substitutes for the forest under consideration it may be reasonable to assume constant and given shadow prices, which could be estimated by contingent valuation methods.

(iii) However, quite often it is not possible to find "substitute" forests. It may still be possible to argue that there exists a vector of constant shadow prices which would approximately generate the same management plan as would the willingness to pay function. These are the steady state prices.

If we let the planning horizon approach infinity, it is easily seen that there may be two different steady states. The first one is a state with no cutting and with all trees in the highest age class n. This will also be the optimal steady state if the willingness to pay for environmental services is such that old trees are very highly valued compared with younger trees.

The other possible steady state is the normal or synchronized forest. It corresponds to the existence of an integer $N' \leq n$ such that trees of age less than N' are not cut, while all trees of age N' are cut. This would correspond to a situation where the general public wants a variety of stands with trees in different age classes. It is this steady state that will be considered below.

The normal forest is a traditional concept in forest economics, and it would not be unfair to say that foresters have had an intuitive feeling for the potential asymptotic optimality of the normal forest. The formal theorems are, however, very recent. Mitra and Wan (1986) show the following theorem.

Theorem: When future profits are undiscounted, and the profit function is strictly concave in the cut, the optimal program from any initial situation will converge to a unique optimal stationary program (the normal forest). The yearly cut along the optimal stationary program will be equal to the maximum sustainable yield.

In other words, if the interest rate is zero and the marginal profit (utility) of the last cubic meter of the harvest is decreasing, the composition of the forest land will eventually reach the normal forest. The intution behind the result is to some extent clarified by Figure 1.

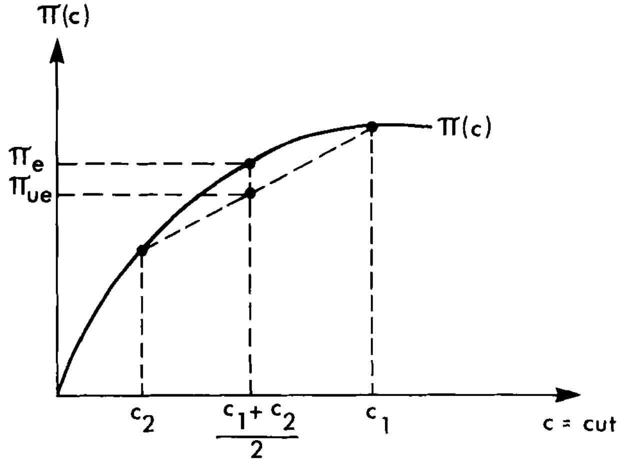

Figure 1. Cutting patterns under a strictly concave profit function.

Let c_1 and c_2 be the harvests today and tomorrow, respectively, under an uneven cutting pattern generated by a nonuniform age distribution. Moreover, let $\bar{c}_{12} = (c_1 + c_2)/2$ represent the yearly cut under an even cutting pattern resulting from a uniform age distribution. Obviously, the uniform cutting pattern is preferred ($\pi_e > \pi_{ue}$) to the nonuniform, indicating that it pays to gradually smooth the age distrbution of the forest.

Loosely speaking, when profits are undiscounted the total gain from a transformation to a more smooth cutting pattern is unbounded, while the total loss along a path towards the normal forest is bounded. No wonder that the solution converges towards the normal forest. It can, however, be shown by a counter example that the above theorem is no longer valid if profit is discounted[21]. The intuitive reason is that a positive interest rate means that both the gain from the normal forest and the loss from a path towards it are bounded. If the adjustment loss is higher than the gains from smoothness, there will be no convergence towards the optimal stationary program.

An analoguous theorem can be proved in a case when there are recreational values present. What has to be done is to reinterpret the production function so it includes the production of not only commercial values, but also recreational values. In other words, the results in Mitra and Wan (1985, 1986) are transferable, although not directly, to our formulation of the problem. See Appendix.

Hence, if the interest rate is "small enough" one might conjecture[22] that the solution for $T \to \infty$ will converge to a unique normal forest $\bar{x}(q, \bar{\alpha}, r)$, and a unique vector of

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21 See Mitra and Wan (1985).

22 The conjecture is supported by e.g., numerical analyses in Kemp and Moore (1979), but has not yet been formally proved.

pseudoequilibrium prices $\overline{\alpha}$. Given that we by, say, contingent valuation methods, can find the relevant steady state pseudoequilibrium prices (or a good approximation of them) we can use these in the origional problem together with timber prices and the interest rate, and hope that the problem, so modified, will converge to the same steady state. Clearly, if the steady state is unique as a function of prices and the interest rate, a convergence as such would do the job. We would, however, need information on the rotation period in the (maximum sustainable yield) steady state. Accurate quantitative information is of course difficult to obtain, but there are qualitative information on the shape of the normal forest in the presence of environmental services. More precisely, if the environmental benefits are increasing, constant or decreasing as a function of the age of the stand, then the commercial steady state rotation will be shorter, equal to or longer than the rotation period that solves the problem in the presence of environmental services (the socially optimal steady state rotation period).

A non-technical "proof" runs like this: if the environmental yearly benefit is independent of the age of the standing trees – say equal to a constant V_0 – its present value equals the value of a perpetuity yielding V_0 dollars/year, or V_0/r, where r is the interest rate. Clearly this expression is independent of the rotation age, and the commercial rotation period coincides with the socially optimal rotation period. If the environmental benefits increase with the age of the stand, the extra value created by older trees can only be realized by a lengthening of the commercial rotation. The opposite is obviously true if the benefits are decreasing with the age of the trees.

For a formal proof see Hite et.al. (1987) or Johansson and Löfgren (1988a), where it is easily seen that the above result holds also for the case when $r = 0$.

5 CONCLUSION

The intention of this paper is to apply results from the theory of Lindahl equilibria in economies with public goods to the multiple use management of public (and private) forest land. We show that, given that we can treat the environmental services of standing trees as public goods and that we know the general equilibrium prices for public goods, we can then decentralize the social optimization problem by adding an environmental component to the ordinary present value problem. One point being that this component is linear in the case of stand variables, and the problem that the valuation of one stand depends on the states of others is automatically solved by the general equilibrium prices.

We use the theoretical knowledge on the existence of these prices in a microeconomic analysis of the multiple use management problem. The results are far from surprising, since they are analogous to similar results from neoclassical theory, but are, nevertheless, worth stating.

A very difficult and essentially unsolved practical problem is to find the pseudoequilibrium prices. We suggest three "methods" to approximate these prices; the third being the most interesting. Here we use the theoretical convergence properties of the management problem to support an idea that people should be asked about how they value stands in a regulated forest. These valuations can then be used to generate a cutting pattern not too far from the "first best optimum". Under ideal conditions the cutting pattern would asymptotically converge to the optimal steady state.

Some of the forests which should be completely saved from the start – such as virgin forests – would probably be possible to pinpoint by less subtle methods.

APPENDIX

Let f(t) be the production function of a stand on one acre of land, where t denotes time. Moreover, let $g(s) \geq 0$ for $s \geq 0$ be the environmental benefit function of a stand of age s. Over a short interval ds, the total environmental benfit is g(s)ds, and its present value is $e^{-rs}g(s)ds$ (this entity corresponds to our α_{ti}^{r}:s, and {f(1), f(2), ..., f(n)} corresponds to our vector g).

When the interest rate is zero the Hartman problem can be shown to transform into the maximization of the sustainable combined yield, i.e.,

$$\max_t \frac{1}{t}\left[f(t) + \int_0^t g(s)ds\right]$$

If we define

$$f(t) + \int_0^t g(s)ds = F(t)$$

and

$$H(t) = F(t)t^{-1}$$

all the results in Mitra and Wan (1986) will follow, if we invoke on F(t) and H(t) all the properties of the "Mitra and Wan"–commercial production function. In particular, the optimal program will, from any initial state, converge toward a normal forest, if the production function is strictly concave in the cut. Note that the cut is now valued both for its timber content, and its accumulated environmental values.

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