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Chapter 9

DISEQUILIBRIUM COST-BENEFIT RULES: AN EXPOSITION AND EXTENSION*

PER–OLOV JOHANSSON and KARL-GUSTAF LÖFGREN
Department of Forest Economics, SLU, S–901 83 Umeå, (Sweden)
and
FIEF, Wallingatan 38, 4 tr, S–111 24 Stockholm, (Sweden)

1 INTRODUCTION

This paper reviews some of the recent developments in disequilibrium cost-benefit analysis. Instead of repeating previous modelling attempts we generalize some of the results by deriving cost-benefit rules within an intertemporal multi-sectoral model of a small open economy, with endogenous private investment. We show in particular how some of the earlier results follow as special cases.

Distributional issues are most often neglected by focusing on efficiency considerations, and introducing a representative household. To our knowledge the distributional problems have so far not been addressed in a disequilibrium cost-benefit setting. A special section below is devoted to a discussion of necessary and sufficient conditions for welfare improvement in the compensation sense.

The paper is structured as follows: After a short survey of the literature we introduce our general equilibrium model and show how the general equilibrium cost-benefit rules follow by differentiation of the indirect utility function of the representative household with respect to the project parameter. Next, we derive the corresponding cost-benefit rules for two typical disequilibrium situations; classical and Keynesian unemployment. We devote a special section to a discussion of cost-benefit rules for natural resource projects, and the section on distributional issues concludes the analytical part of the paper.

2 A BRIEF SURVEY OF THE LITERATURE

The beginnings of cost–benefit analysis date back over a century to the work of Jules Dupuit, who was concerned with the benefits and costs of constructing a bridge; Dupuit's famous paper "On the Utility of Public Works" was published in 1844. In particular, Dupuit introduced the concept of the consumer's surplus, i.e. the fact that benefits are measured by an area under a demand curve, not by what is actually paid. The next major contribution to cost–benefit analysis seems to have appeared almost one hundred years later.

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In a well-known paper, "The General Welfare in Relation to Problems of Railway and Utility Rates" published in 1938, Harald Hotelling, among other things, formulated the case for marginal cost pricing: "The efficient way to operate a bridge is to make it free to the public, so long at least as the use of it does not increase to a state of overcrowding" [Hotelling (1938, p. 158)].

The first attempts to apply cost–benefit analysis to empirical decision making also date back to the thirties. The United States Flood Control Act of 1936 introduced the principle that a project is desirable if "the benefits, to whomsoever they may accrue, are in excess of the estimated costs". However, the precise meaning of a "benefit" remained unclear, and individual agencies often approached similar projects from different standpoints. This stimulated academic interest in the subject. In particular, beginning in the late fifties, an extensive literature on the foundations of cost–benefit analysis emerged; see Eckstein (1958), McKean (1958), Krutilla and Eckstein (1958), Maass (1966), Marglin (1967), Harberger (1969, 1971), Musgrave (1969), Lesourne (1975), Little and Mirrlees (1968), Dasgupta et al (1972), Boadway (1975), Srinivasan and Bhagwati (1978), and Diewert (1983), just to mention a few.[1]

Several of these works can be considered as manuals since they are concerned with many problems such as the choice of an appropriate discount rate, measuring the opportunity cost of capital withdrawal from the private sector, and the efficiency gains from public sector projects in a tax–distorted economy. Of particular interest in the present context, however, is the treatment of market imbalances in the aforementioned as well as in later contributions.

For a long period of time interest has been focused on disequilibria in the labor market. The traditional way to view employment–creating (public–sector) projects is highlighted by the following quotation from Musgrave and Musgrave (1973, p. 161): ".... employment effects of particular projects become relevant to benefit evaluation if alternative policies to deal with unemployment are not available. The resulting gain in employment is then an additional benefit, or the opportunity cost of labor is zero". This well–known partial equilibrium view found in most textbooks is illustrated in Figure 1, where the wage is fixed above the market–clearing level. Obviously, all labor employed in a marginal project can be treated as coming from the unemployed. Therefore, one should attribute a positive shadow price to the labor hired only to the extent that households perceive disutility from additional employment. (A zero disutility from work effort produces the rule or result quoted above.)

Given this partial equilibrium view, it is perhaps not too surprising that empirical studies often include multiplier effects. The idea behind this is that incomes earned by the formerly unemployed people are spent, initiating the multiplier process well–known from textbooks on Keynesian macroeconomics. This is, of course, equivalent to assuming that there is excess capacity in some sectors of the economy, i.e. unemployment is due to deficient demand.

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[1] Arrow and Kurz (1970), Meade (1955), and Tinbergen (1952) are examples of related works which have influenced the development of cost-benefit analysis.

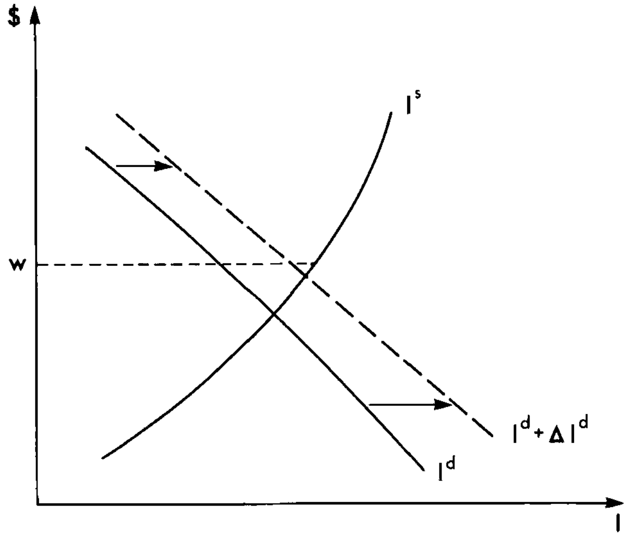

Figure 1. The partial equilibrium view on the employment effect of an increased demand for labor when the wage rate is fixed above the market–clearing level.

However, for a long period of time, cost–benefit analysts had difficulty in providing a theoretical justification for their treatment of the macroeconomic effects caused by a project. This difficulty arose because of the lack of a satisfactory link between microeconomics and Keynesian macroeconomics. Whereas public finance theory uses models based on individual optimization, it is ill–equipped to deal with non–market–clearing situations which lie beyond its Walrasian equilibrium framework. Macroeconomics, on the other hand, focuses on market imbalances but its microeconomic underpinning has often been weak. This lack of microeconomic foundations has made it difficult to directly assess the welfare effects of government policies.

In their seminal paper, "A General Disequilibrium Model of Income and Employment", Barro and Grossman (1971) provided a link between microeconomics and Keynesian macroeconomics. However, to the best of our knowledge, this link was not discovered by cost–benefit analysts until the early eighties. It turns out that the general disequilibrium approach in general produces cost–benefit rules which are very different from the partial equilibrium rules presented above. There are at least two generations of models that have been used in deriving disequilibrium cost–benefit rules and in making welfare evaluations. The early papers, e.g. Bell and Devarajan (1983), Blitzer et al (1981), Cuddington et al (1984), J.H. Dreze (1985), J.P. Dreze (1982), Fourgeaud et al (1986), Johansson (1982), Maneschi (1985) and Roberts (1982), use essentially single period models with exogenous private investment. This means that a public–sector project has no adverse effect on the level of investment. The second generation models are intertemporal, which admit an explicit treatment of expectations, and some of them treat private investment as an endogenous variable; see e.g. Johansson (1984), Johansson and Löfgren (1985), and Marchand et al (1984, 1985).

Instead of reviewing this literature, we will present a model similar to the one in Neary and Stiglitz (1983) which, as other special cases, generates most, if not all, of the results found in the aforementioned literature. In addition, this approach will enable us to derive some results which we believe are new.

3 THE MODEL IN THE ABSENCE OF QUANTITY CONSTRAINTS

This section considers a small open economy which can buy and sell tradeables without limit in each period at fixed foreign currency prices. Moreover, assuming perfect capital mobility and a single international traded bond, the foreign-currency interest rate is exogenously fixed at the world level for the small open economy under consideration. The exchange rate, which translates foreign-currency prices into domestic currency prices, is assumed to be fixed. This assumption may seem unduly primitive given the recent developments of balance of payment theory. However, the cost-benefit rules derived below are robust with respect to the treatment of the (flexible) exchange rate mechanism. The price of non-traded goods and the wage rate are allowed to adjust so as to achieve equilibrium in the market for non-tradeables and the labor market, respectively.

3.1 Production sectors

Within this environment, private firms maximize the present value of profits given prices and the production technology. The profit function, or perhaps better the present value function, is written as:

$$\Pi^i(p^i, w, \odot^i) = \max_{\ell^i, I^i} \{p^i(y^i - I^i) - w\,\ell^i + \phi^i(I_2^i, \odot^i) \mid y_1^i = F^i(\ell_1^i),\ y_2^i = H^i(\ell_2^i, I_1^i)\} \tag{1}$$

where a superscript i = x refers to producers of traded goods and i = n refers to producers of non-traded goods, $p^i = (p_1^i, p_2^i)$ is a vector of prices containing the present value price in period t (t = 1, 2), w is a vector of present value wage rates, y^i is a vector of gross outputs of goods, I^i is a vector of investments, ℓ^i is a vector of demands for labor, $\phi^i(\cdot)$ denotes profits in all periods beyond the second period as a function of the level of investment in the second period and expectations $\odot^i$ of future prices, etc, $F^i(\cdot)$ and $H^i(\cdot)$ are the twice continuously differentiable and strongly concave first-period and second-period production functions, respectively, all prices are domestic currency prices, and notation denoting transposed vectors have been suppressed.

According to (1), firms choose current and future employment levels. In addition, firms may invest part of the output in order to augment productivity of labor in the future. Note that we "collapse" all periods beyond the second period into a single period so that $\phi^i(\cdot)$ contains the sum of the present values of expected profits in these future periods.

In later sections, we will employ the envelope theorem in order to simplify the exposition; see e.g. Varian (1984) for details. For example, the effect of a ceteris paribus change in p_1^i on the present value of profits is equal to:

$$\partial \Pi^i(p^i, w, \Theta^i)/\partial p_1^i = y_1^i(p_1, w_1) - I_1^i(p, w_2) = x_1^i(p, w) \tag{2}$$

where $x_1^i(\cdot)$ is the first-period supply of commodity i, i.e. what is produced less what is invested of the commodity. Thus, by taking the partial derivative of (1) with respect to a price, we obtain the net supply of that commodity.

3.2 The Government

To derive cost-benefit rules we will follow the tradition within the field and introduce (two) state-owned firms. These firms produce traded goods and non-traded goods, respectively, using labor as the sole variable input. Any profits (or losses) incurred by the state-owned firms are assumed to be disposed of (financed) by lump-sum transfers (taxes) T. Hence, the government budget constraint takes the form:

$$T = \sum_{i=x}^{n} [p^i y_g^i - w\,\ell_g^i] \tag{3}$$

where the g subscript denotes government output supply or labor demand, i.e. $y_g^i = (y_{g1}^i, y_{g2}^i)$, $\ell_g^i = (\ell_{g1}^i, \ell_{g2}^i)$, and $y_{gt}^i = f_t^i(\ell_{gt}^i)$. State-owned firms hire labor at the prevailing wage and sell output at the prevailing prices, i.e. these firms, like all other agents in the economy, are assumed to take prices as given. Because the level of public sector employment, by assumption, is an exogenously-determined policy variable, the marginal revenue product of government-employed labor may exceed, be equal to, or fall short of the wage; $p_t^i(\partial y_{gt}^i/\partial \ell_{gt}^i) \gtreqless w_t$ for i = x, n, t = 1, 2.

This simple specification of the public sector allows us to concentrate on the market imbalance issue in later sections. In addition, it is the specification used in almost all papers deriving (disequilibrium) cost-benefit rules. See for example the references introduced above.

3.3 Households

In order to focus on efficiency considerations while setting aside matters of equity and income distribution, the commonly employed assumption of a "representative" household is used in Sections 3 – 5, but in Section 6 we discuss distributional issues in terms of the Kaldorian compensation criterion, see Kaldor (1939). The household consumes both traded and non-traded goods and supplies labor. In addition, the household is assumed to save in order to be able to consume in periods beyond the second period. This is captured by including as an argument in the utility function holdings of an asset at the end of the second period.

Both borrowing and lending are allowed at the prevailing interest rate. Once the possibility of borrowing or lending at the prevailing interest rate is introduced, the issue of whether profits are distributed in the period in which they are generated or in the subsequent period becomes less important. In what follows it will be assumed that the sum of current profits from both private producers and state owned firms are distributed within the current period.

The household is assumed to maximize utility subject to its budget constraint. The indirect utility function of the household is defined as:

$$\nu(p, w, \Pi + T + M_0, \odot) = \max_{X, \ell, M} \{U(X, \ell, M, \odot) \mid M_0 + \Pi + T + w\ell - pX - M = 0\} \tag{4}$$

where $p = (p^x, p^n)$ is a vector of goods prices, $\Pi = \Pi^x + \Pi^n$, M_0 is initial wealth yielding the international interest rate, $\odot$ denotes expectations of prices in periods beyond the second period, $U(\cdot)$ is the twice continuously differentiable and strongly quasi–concave utility function, $X = (X_1^x, X_2^x, X_1^n, X_2^n)$ is a vector of demands for goods, $\ell = (\ell_1, \ell_2)$ is a vector of supplies of labor, and M is end of second period wealth.

According to (4), (indirect) utility is a function of all prices and wages, exogenous income, and expectations. This indirect utility function has all the properties known from textbooks on microeconomics. For example, taking the partial derivative with respect to a goods price, one obtains the corresponding demand function (multiplied by $-\lambda$, where λ denotes the marginal utility of exogenous income Y, i.e. $\partial\nu/\partial Y$).

3.4 Society's welfare function

From the point of view of the entire economy, the profits of both private sector and public sector firms in equation (4) are functions of prices and wages, i.e. they cannot be treated as exogenous entities as in (4). In addition, public sector profits are determined by the levels of employment ℓ_g^i since these are exogenously determined policy variables. Therefore, using (4) the indirect utility function, or the social welfare function in our single household economy, can be written as:

$$\nu(p, w, Y + M_0, \odot) = V(p, w, \ell_g, M_0, \Theta) \tag{5}$$

where $Y = \sum_{i=x}^{n} \Pi^i(p^i, w, \odot^i) + \sum_{t=1}^{2} \sum_{i=x}^{n} [p_t^i f^i(\ell_{gt}^i) - w_t \ell_{gt}^i]$, $\ell_g = (\ell_g^x, \ell_g^n)$ is a vector of government labor demands, and $\Theta = (\odot, \odot^x, \odot^n)$.

Taking the partial derivative of (5) with respect to p_t^i, one obtains:

$$\partial\nu/\partial p_t^i + (\partial\nu/\partial Y)(\partial Y/\partial p_t^i) = \partial V/\partial p_t^i = -\lambda X_t^i + \lambda(x_t^i + y_{gt}^i) \qquad \forall\, i, t \tag{6}$$

since $\partial\nu/\partial p_t^i = -\lambda X_t^i$, $\partial\nu/\partial Y = \lambda$, $\partial Y/\partial p_t^i = \partial \Pi^i/\partial p_t^i + \partial T/\partial p_t^i$, $\partial \Pi^i/\partial p_t^i = y_t^i - I_t^i = x_t^i$, and $\partial T/\partial p_t^i = y_{gt}^i$.

Thus, by taking the partial derivative of (5) with respect to a price (or the wage rate), one obtains the excess supply or demand of the commodity in question (multiplied by λ). Obviously, if all prices and wages correspond to their market clearing levels, then $\partial V/\partial p_t^i = 0$ in (6) for all i and t. This is a useful result which will simplify considerably the derivations of the cost-benefit rules.

4 GENERAL EQUILIBRIUM AND DISEQUILIBRIUM COST-BENEFIT RULES

To obtain a monetary welfare change measure of a marginal change in the public project, we differentiate totally the welfare function (5) with respect to ℓ_{gt}^i and divide through by λ, to obtain:

$$(dV/d\ell_{gt}^i)/\lambda = p_t^i(\partial y_{gt}^i/\partial \ell_{gt}^i) - w_t \qquad \forall\, i, t \tag{7}$$

Dividing through by λ, the marginal utility of exogenous income, converts the right-hand side expression from unobservable units of utility to observable units of money. According to the monetary welfare change measure (7), the appropriate rule for project evaluation would be to value all outputs and inputs at their domestic market-clearing (present value) prices. Using this criterion, it is clear that welfare can be increased as long as the marginal revenue product of government employed labor differs from the wage rate. This is the well-known general equilibrium rule found in the literature on cost-benefit analysis. See e.g. Boadway (1975), Harberger (1971), Lesourne (1975), and Starrett (1979).

Note that a change in the level of public sector employment may affect some or all of the economy's prices. However, given the assumption of continuous market-clearing prices, equation (6) ensures that any indirect effects through changed prices are equal to zero (or rather the net effect is zero). Therefore, the measure (7) is a general equilibrium measure, although it is only valid for "small" changes in ℓ_{gt}^i. For the problems of examining discrete changes, the reader is referred to Johansson (1987), Starrett (1979), and Tsuneki (1985).

The assumption of continuous market-clearing prices employed above in deriving cost-benefit rules is a strong one. For practical applications it is invaluable to determine how the shadow pricing rules are changed by different kinds of market imbalances. In this section it is shown how the model can be used to derive project evaluation or cost-benefit rules for situations where there is quantity rationing due to price stickiness in markets for goods and factors[2]. Since the model contains several commodities and periods, it is possible to construct

[2] For a survey of different reasons for sticky prices, the reader is referred to Cuddington et al (1984) and Stiglitz (1986).

a lot of disequilibrium variations, but to keep the problem tractable we will only consider two cases. The basic idea is to make the important distinction between the short run (period 1) and the medium run or long run (period 2). In the short run, the wage rate is assumed to be sticky. However, in the long run it adjusts to its market–clearing level. In our opinion, this is a scenario which is close to the one often faced by policy makers, i.e. the conflict between short–run targets such as a low level of unemployment and medium or long run targets such as a high level of investment[3] (growth).

Two typical situations are considered. The first situation, labelled classical unemployment, is characterized by (first period) excess supply in the labor market due to excessive real wages (and a trade balance deficit or surplus due to a fixed exchange rate). The other situation, called Keynesian unemployment, refers to a situation where there is unemployment due to deficient demand. Consider first classical unemployment.

4.1 Classical unemployment

The fact that there is an excess supply of labor, but no other market imbalances (quantity constraints), means that firms are still unconstrained in all markets. Hence, the maximization problems of firms are those described in Sections 3.1 and 3.2 above. On the other hand, the household maximizes its utility function subject to the budget constraint plus the first period employment constraint $\ell_1 = \bar{\ell}_1$.

Straightforward but tedious calculations verify that the indirect utility function (5) now can be written as:

$$\hat{\nu}(p, w_2, Y + w_1\bar{\ell}_1 + M_0, \bar{\ell}_1, \odot) = \hat{V}(p, w_2, \ell_g, w_1\bar{\ell}_1 + M_0, \bar{\ell}_1, \Theta) \tag{8}$$

where $\bar{\ell}_1 = \ell_1^x + \ell_1^n + \ell_{g1}^x + \ell_{g1}^n$, i.e. the household does the best possible and sells the amount of labor demanded. The main difference between the indirect utility function in (8) and the one in equation (5) is that a change in the first–period wage rate now has an income effect, but not a substitution effect. This is so because the household faces a binding constraint in the labor market in period 1, i.e. the household cannot adjust the level of employment following a change in the wage rate.

Consider now a small change in the level of employment in period 1. Taking the partial derivative of (8) with respect to $\bar{\ell}_1$, one obtains:

$$\partial\hat{V}/\partial\bar{\ell}_1 = \hat{\lambda}w_1 + \hat{U}_\ell \tag{9}$$

[3] In contrast, in the intertemporal model used by Marchand et al (1985) to derive shadow prices there is no investment or production of goods in period 2, i.e. all such activities take place in period 1 (but the goods become available for consumption in period 2).

where all prices and public sector output levels are being held constant, and $\hat{U}_{\bar{\ell}_1}$ refers to the partial derivative with respect to the next to last argument of the indirect utility function (8). In fact, $\hat{U}_{\bar{\ell}_1}$ is the marginal disutility of work effort, evaluated at the optimal or utility maximizing levels of all unconstrained household demands and supplies. In the literature, $\hat{U}_{\bar{\ell}_1}/\hat{\lambda}$ is sometimes referred to as (the negative of) the supply price or the virtual price of labor[4]. Equation (9) reflects the fact that there is a difference between the market wage and the marginal disutility of work effort whenever the household is unemployed/underemployed (or, for that matter, overemployed).

In order to obtain cost–benefit rules to be used in the case of classical unemployment, let us assume that first–period public sector employment and production are increased. Taking the total derivative of (8) with respect to ℓ^i_{g1} and dividing through by $\hat{\lambda}$ yields:

$$(d\hat{V}/d\ell^i_{g1})/\hat{\lambda} = p^i_1(\partial y^i_{g1}/\partial \ell^i_{g1}) - w_1 + (w_1 + \hat{U}_{\bar{\ell}_1}/\hat{\lambda})(\partial \bar{\ell}_1/\partial \ell^i_{g1}) \tag{10}$$

$$= [p^i_1(\partial y^i_{g1}/\partial \ell^i_{g1}) + (\hat{U}_{\bar{\ell}_1}/\hat{\lambda})] + [(w_1 + \hat{U}_{\bar{\ell}_1}/\hat{\lambda})(\partial \ell^p_1/\partial \ell^i_{g1})]$$

where $\ell^p_1 = \ell^x_1(p^x_1, w_1) + \ell^n_1(p^n_1, w_1)$, i.e. ℓ^p_1 denotes private sector employment in period 1.

The welfare measure in the middle expression in (10) differs from the one obtained in the general equilibrium case in that a new term reflecting the policy induced change in total employment (private plus public) is now added. This term reflects the fact that the marginal disutility of effort is less than the sticky nominal wage whenever there is unemployment.

The middle expression in equation (10) provides a straightforward rule for evaluating public sector enterprises under classical unemployment. First, evaluate all outputs and inputs of state–owned enterprises at prevailing domestic market prices and assess the firms' profitability on this basis. Second, determine the total policy induced change in employment in both the private and public sectors ($d\bar{\ell}_1$) and evaluate this change at the prevailing wage less any adjustment reflecting household disutility of increased work effort.

Let us compare this rule with the traditional partial equilibrium rule found in textbooks and briefly discussed in Section 2. The partial equilibrium view, illustrated in Figure 1 in Section 2, treats all labor employed in a marginal project as coming from the unemployed, and implicitly ignores any effect of increased public sector employment on employment in the private sector. Therefore, the partial equilibrium rule corresponds to the first expression within brackets in the right–hand side expression of equation (10). This says: Value all public sector output at market prices but attribute a positive shadow price to the labor hired only to

[4] See Cuddington et al (1984), Neary and Roberts (1980), and Rothbarth (1940/41). $\hat{\lambda}$, the marginal utility of money, transforms money to units of utility.

the extent that households perceive disutility from additional employment (i.e. $\hat{U}_{\ell_1}^- < 0$).

Unfortunately, this shadow pricing rule based on partial equilibrium analysis is in general incorrect even though the public sector hires only from the pool of unemployed workers. The reason for this is that even a marginal increase in public sector employment will affect prices and hence private sector demand for labor. This indirect dependence is captured by the second expression within brackets in equation (10). For example, if total employment decreases, i.e. $d\ell_{g1}^i + d\ell_1^p < 0$, the presence of the last expression within brackets in (10) implies that profitability calculated at producer prices – which is the general equilibrium rule derived in equation (7) above – is not sufficient to ensure that a project is socially profitable. According to the partial equilibrium rule, on the other hand, a project is always socially profitable if it is profitable when measured at producer prices.

Consider next changes in future output levels. Suppose that the government announces that public–sector production of nontradeables will increase in the future, i.e. in period 2. Since all period 2 prices are allowed to adjust to their market–clearing levels, it is quite natural to use equation (7) to assess the profitability of the project in question. However, in the present context, where there is unemployment in period 1, this is a partial or incomplete rule.

To show this, note that the increased supply of nontradeables in period 2 will drive down the equilibrium price p_2^n of such goods. In turn, this induces private sector firms to reduce investment and hence gross production of non–traded goods in period 1; recall that less capacity is needed in period 2 due to the decrease in p_2^n. Therefore, first period employment is reduced, thereby imposing an extra cost on the considered project. This extra cost is captured by an expression similar to the final expression within brackets in equation (10), and should be added to the project evaluation rule (7), which is valid if there is full employment in all periods.

These results show that it is of utmost importance for the policy maker to try to estimate and include also the indirect or induced effects caused by a particular project. Otherwise, the policy maker may implement projects which are socially unprofitable and reject projects that are socially profitable. In particular, it shows that relief work announced to be conducted during future periods of full employment have effects in the current unemployment period.

4.2 Keynesian unemployment

If both prices and wages are fixed such that there is excess supply of both goods and labor, one speaks of Keynesian unemployment. In this paper, however, the analysis is restricted to a situation where there is unemployment and private producers of non–traded goods face a sales constraint for their output in the short–run, i.e. in period 1. Firms producing traded goods are throughout assumed to never face quantity constraints. As before, in the long run, represented by period 2, prices and wages are flexible so that neither households nor firms are rationed.

Private firms producing non–traded goods face a sales constraint $\bar{x}_1^n = y_1^n - I_1^n$. Adding this constraint to the maximization problem (1) for i = n, yields a profit function which can be written as:

$$\hat{\Pi}^n(p^n, w, \bar{x}_1^n, \odot^n) = \Pi^n(\hat{p}_1^n, p_2^n, w, \odot^n) + (p_1^n - \hat{p}_1^n)\bar{x}_1^n \tag{11}$$

where p_1^n is the market price of non–traded goods in period 1, and $\hat{p}_1^n$ is the virtual price of such goods in period 1. This virtual price is such that the quantity unconstrained supply x_1^n is equal to the quantity constraint $\bar{x}_1^n$ (and all other supply and demand levels remain unchanged). Obviously, since the firm is rationed, then $p_1^n > \hat{p}_1^n$. This implies that the right–hand side profit function in (11) yields a lower level of profits than the left–hand side profit function, although $\hat{p}_1^n$ is chosen in such a way that both functions produce the same optimal supply and demand levels. This explains the fact that we add the expression $(p_1^n - \hat{p}_1^n)\bar{x}_1^n$ to the right–hand side of (11).

Taking the partial derivative of (11) with respect to $\bar{x}_1^n$, one obtains:

$$\partial\hat{\Pi}^n/\partial\bar{x}_1^n = (\partial\Pi^n/\partial\hat{p}_1^n)(\partial\hat{p}_1^n/\partial\bar{x}_1^n) - \bar{x}_1^n(\partial\hat{p}_1^n/\partial\bar{x}_1^n) + (p_1^n - \hat{p}_1^n) = (p_1^n - \hat{p}_1^n) > 0 \tag{12}$$

where $\partial\Pi^n/\partial\hat{p}_1^n = x_1^n(\hat{p}_1^n, p_2^n, w) = \bar{x}_1^n$. According to (12), the increase in profits caused by a small increase in sales is equal to the difference between the ruling market price and the virtual price of nontradeables, as is illustrated in Figure 2a.

Roughly speaking, substituting (11) into (8) we obtain the social indirect utility function to be used in Keynesian unemployment situations. Taking the total derivative of this function with respect to the policy parameter and dividing through by the marginal utility of income, we obtain the cost–benefit rule to be used in the presence of Keynesian unemployment:

$$(d\tilde{V}/d\ell_{g1}^i)/\tilde{\lambda} = p_1^i(\partial y_{g1}^i/\partial\ell_{g1}^i) - w_1 + (p_1^n - \hat{p}_1^n)(\partial\bar{x}_1^n/\partial\ell_{g1}^i) + (w_1 + \tilde{U}_\ell/\tilde{\lambda})(\partial\bar{\ell}_1/\partial\ell_{g1}^i) \qquad i = x, n \tag{13}$$

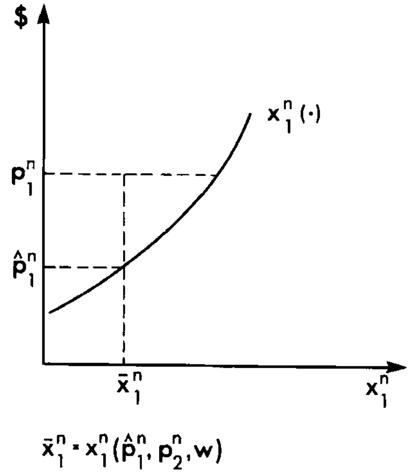

Figure 2a. Illustration of the definition of a virtual price ($\hat{p}_1^n$).

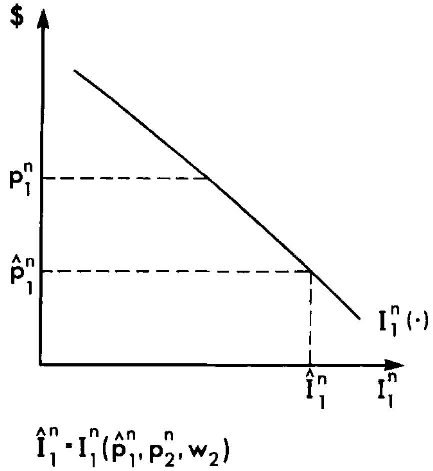

Figure 2b. Illustration of the firm's investment function.

Note that (13) reduces to (10) if nontradeables firms are not rationed since then $p_1^n = \hat{p}_1^n$ in (13). However, given that the total private and public sector production of nontradeables is now demand determined, an increase in public sector employment in the firm producing non–traded goods causes a redistribution of production between the privately owned and the state–owned firms (assuming that the public sector firm gets priority in the market for non-traded goods).

In order to further interpret (13), let us consider increased government production of non–traded goods. There are at least two differences between the project evaluation rule (13) for i = n and the rule for project evaluation under Keynesian unemployment found in earlier works, e.g. Cuddington et al (1984) and Johansson (1982). In earlier works[5], based on single–period models with exogenous investment, there is a one–to–one relationship between the decrease in the level of production of privately owned firms and the level of increase of production of non–traded goods by the government, i.e., $dy^n_{g1} + d\bar{x}^n_1 = 0$ (assuming that the household utility function is weakly separable in first period labor supply, an assumption which therefore is implicitly employed also here). Therefore, any change in welfare must follow from differences, if any, in the marginal productivities of labor between privately owned and state–owned firms. Welfare will increase only if state–owned firms are more efficient than private sector firms so that total employment decreases (provided that $\tilde{U}_\ell < 0$ as is usually assumed). This conclusion can be arrived at by setting $dx^n_1 = - dy^n_{g1}$ and $dI^n_1 = 0$ in (14) below, but see e.g. Cuddington et al (1984) for a detailed derivation of this result.

However, in the model used in this paper, private sector firms are allowed to adjust their levels of investment following a decrease in sales. In fact, in the appendix at the end of the paper it is shown that demand–constrained firms increase investment if sales, i.e. $\bar{x}^n_1$, decreases. The reason is, loosely speaking, that the opportunity cost of investment increases when the demand constraint is relaxed. This result, which is illustrated in Figure 2b, questions the popular belief that measures aimed at reducing unemployment in the short run tend to crowd–out private investment. In turn, this change in private investment affects employment and hence the household's income in such a way that final demand for non–traded goods increases. However, private sector firms sales are still reduced, but there is not complete crowding–out as in earlier models, i.e. $0 > \partial\bar{x}^n_1/\partial y^n_{g1} > - 1$ in our model. (In arriving at this result we have suppressed any induced changes in p_2 and w_2.) Therefore, our project evaluation rule, stated in (13), is somewhat less discouraging than the one found in e.g. Cuddington et al (1984) and Johansson (1982). Nevertheless, even in our model, a decrease in total employment is a sufficient condition for welfare to increase following an increase in government production of non–traded goods when the economy suffers from Keynesian unemployment.

To prove this claim, equation (13) is written in the following way:

$$(d\tilde{V}/d\ell^n_{g1})/\tilde{\lambda} = p^n_1[(\partial y^n_{g1}/\partial\ell^n_{g1}) + \underset{(+)}{(\partial\bar{x}^n_1/\partial\ell^n_{g1})}] + \hat{p}^n_1\underset{(+)}{(\partial I^n_1/\partial\ell^n_{g1})} + \underset{(-)}{(\tilde{U}_\ell/\tilde{\lambda})}\underset{(?)}{(\partial\bar{\ell}_1/\partial\ell^n_{g1})} \qquad (14)$$

where we have used the fact that $\hat{p}^n_1 d\bar{x}^n_1 = \hat{p}^n_1[\partial F^n(\ell^n_1)/\partial\ell^n_1]d\ell^n_1 - \hat{p}^n_1 dI^n_1 = w_1 d\ell^n_1 - \hat{p}^n_1 dI^n_1$; recall that $\hat{p}^n_1$ is such that a profit maximizing firm would supply exactly $\bar{x}^n_1$ if unconstrained

[5] We refrain from a comparison with Marchand et al (1985) since, in their intertemporal model with endogenous investment, all production activities take place in period 1. Therefore, we believe the models are too different for a meaningful comparison of the results (given a perceived constraint on the acceptable number of pages of this paper).

in this market, implying that the marginal revenue product of labor is equal to the wage, i.e. $\hat{p}_1^n \partial F^n(\ell_1^n)/\partial \ell_1^n = w_1$. Therefore, all "wage terms" net out. The signs below the different terms indicate whether it has a positive or a negative sign; see the discussion above. Obviously, a reduction in aggregate employment ensures that welfare increases following an increase in government production of nontradeables.

A second difference between the cost–benefit rule (13) or (14) and the rule found in earlier works stems from the treatment of expectations. Most previous authors use single–period models where expectations of future prices and quantity constraints are treated as exogenous. In sharp contrast, in our model agents have rational expectations implying that they correctly foresee any changes in second period prices. Therefore, the virtual price of labor as well as private sector demands for labor and investment goods depend on future, i.e. period 2, prices. For example, period 1 demand for labor by private sector firms producing non–traded goods in (14) is $\ell_1^n = \ell_1^n(\bar{x}_1^n, w_1, p_2^n, w_2)$. Since future (market–clearing) price levels may be affected by the considered first period change in government production, the partial derivatives in (14) should be interpreted as including any induced changes in future prices.

In sum, one possible formulation of the project evaluation rule for production of nontradeables under Keynesian unemployment would be: (a) evaluate the net change – private plus public – in the supply of nontradeables at the ruling market price; (b) evaluate any change in investment at the virtual price of nontradeables[6]; and (c) evaluate the net change in total labor demand at the virtual price of labor.

It is of course also possible to use the model to examine the case when a state–owned firm supplies traded goods. Under Keynesian unemployment (and a fixed exchange rate in period 1), increased government production of traded goods in period 1 will generate real income–induced multiplier effects in the nontraded goods sector. Due to the small open economy assumption, private sector firms' supply of traded goods is left unchanged by the considered change in government production, since the relevant relative prices are left unchanged and there is no demand constraint facing firms in this sector. Therefore, national income, i.e. profits plus wage income, increases unambiguously. Part of the new incomes are spent on non–traded goods. Since supply of such goods by assumption is demand–constrained, the usual multiplier process, well–known from textbooks on Keynesian macroeconomics, is initiated.

However, also in this case our model produces slightly different results from those found in previous works. The reason being that private investment is endogenous in our model. Therefore, the cost–benefit rule reads:

$$(d\tilde{V}/d\ell_{g1}^x)/\tilde{\lambda} = p_1^x(\partial y_{g1}^x/\partial \ell_{g1}^x) + [p_1^n + \hat{p}_1^n(\partial I_1^n/\partial \bar{x}_1^n)](\partial \bar{x}_1^n/\partial \ell_{g1}^x) + (\tilde{U}_\ell/\tilde{\lambda})(\partial \bar{\ell}_1/\partial \ell_{g1}^x) \tag{15}$$

[6] Note that today's virtual price of the good in question contains information about conditions tomorrow.

If private investment is unaffected by the change in question, i.e. $\partial I_1^n/\partial \bar{x}_1^n = 0$, then (15) reduces to the "textbook" Keynesian multiplier expression with $p_1^n \partial \bar{x}_1^n/\partial \ell_{g1}^x$ representing income–induced changes – multiplier effects – in the demand–constrained nontradeables sector; see Cuddington et al (1984) for a detailed derivation of the multiplier expression. However, in our model, the magnitude of the income–induced effects is reduced by the decrease in private investment; recall that $\partial I_1^n/\partial \bar{x}_1^n < 0$.

Therefore, to the domestic market value of the direct change in tradeables output one has to add income–induced effects in the demand–constrained nontradeables sector adjusted for the change in investment in this sector (evaluated at the virtual output price). To obtain the welfare cost of increased government production of tradeables, the direct and the income–induced changes in employment should be multiplied by the virtual price of unemployed laborers.

Cost–benefit analysts have always had difficulty in dealing with macroeconomic issues. This difficulty arose because of the lack of a satisfactory link between microeconomics and Keynesian macroeconomics. For example, empirical studies often include real multiplier effects; see e.g. Bohm (1974) and Somers and Wood (1969). Yet these studies are based on the traditional microeconomic model that does not generate real multiplier effects. Equation (15) shows that models of the kind used here have the potential for providing a missing link between microeconomics and Keynesian macroeconomics. It, moreover, provides a theoretical rationale for the above–mentioned practice of including real multiplier effects in cost–benefit analysis (although most, if not all, existing empirical studies neglect the adverse effect on private investment included in the project evaluation rule (15)). Multiplier effects have also often been added in regional project studies. They can sometimes be given a microeconomic foundation through a temporary equilibrium approach. The reader is referred to Johansson (1981) and Ohlsson (1987) for details.

5 NATURAL RESOURCES

The model used in Sections 3–4 does not include natural resources. In this section we briefly discuss cost–benefit rules to be used when evaluating changes in (government) extraction of a non–renewable natural resource such as crude oil or natural gas. The special feature of a non–renewable resource is that the initial stock is given. Therefore, extractors face a constraint over and above the constraints faced by producers of ordinary produced commodities, namely that one cannot produce (extract) more than the initial stock.

The properties of the optimal extraction path of course depend on the exact specification of the extraction technology, etc. However, if there are no costs of extraction, then we have Hotelling's (1931) well–known rule that extraction will take place in periods in which the rate of change of the resource price in current terms equals the market rate of interest. If the price grows at a faster rate it pays to keep the stock intact and accumulate capital gains. A slower rate of change of the price (than the interest rate) induces the extractor to immediately extract the stock and invest the money in the bank. This is so at least if firms are able to sell unlimited quantities at the prevailing (world market) price. The reader interested in further details is referred to e.g. Dasgupta and Heal (1979).

There are, however, important situations where one can reasonably expect quantity constraints in the markets for natural resources. Even if the economy is small in the sense that changes in domestic supply/demand have an imperceptible influence on world prices, domestic producers or consumers may face rationing if the world prices are slow to adjust to eliminate world excess supply or demand. A good example of such a situation would be a small oil–exporting nation facing a world price and sales constraint imposed by the OPEC oil cartel.

Quantity constraints can also arise in situations where the country in question is small in the world market, if it imposes import quotas coupled with domestic price controls. This may result in domestic rationing, yet the country may be small in the sense that it would, in the absence of such distortionary policies, perceive perfectly elastic world supply or demand curves for these products at prevailing world prices.

Even in the absence of import quotas agents may face rationing. The domestic wage may be fixed at a level which, given other prices, results in excess supply or excess demand for labor. This suggests **a priori** that the rules for the optimal management of natural resources in situations with rationing in the labor market are very different from the rules to be used under full employment conditions.

As a point of departure for our discussion, it is assumed that the economy suffers from classical unemployment in the short–run[7]. Then, assuming that there are private profit maximizing extractors of a non–renewable resource as well as a state–owned firm extracting the resource, we obtain the following cost–benefit rule when the state–owned firm increases its level of extraction in period 1 (possibly in order to reduce short–run unemployment):

$$(d\tilde{\tilde{V}}/d\ell_{g1}^{N})/\tilde{\tilde{\lambda}} = (p_1^N - p_2^N)(\partial y_{g1}^N/\partial \ell_{g1}^N) - w_1 - w_2(\partial \ell_{g2}^N/\partial \ell_{g1}^N) + (w_1 + \tilde{\tilde{U}}_\ell/\tilde{\tilde{\lambda}})(\partial \bar{\ell}_1/\partial \ell_{g1}^N) \tag{16}$$

where the N superscript refers to (traded) non–renewable resources. The three first terms in (16) can be interpreted by referring to Hotelling's rule discussed above. In order to maximize profits, the state–owned firm should select a path of extraction such that the marginal profit is equal in both periods. (In the absence of extraction costs, the marginal profit in a period is equal to the present value price p_t^N implying that the first term in (16) produces Hotelling's aforementioned simple extraction rule.) The reader should recall that increased extraction in one period has to be followed by decreased extraction in the second period. Therefore, it is profitable to reallocate extraction and employment between periods as long as the marginal

[7] In the general equilibrium case one derives a cost-benefit rule which exactly corresponds to the present value criterion. The state-owned firm's change in production is profitable provided that the present value of the change in profits is positive.

profit is not equal across all periods (provided such an interior solution is possible and optimal; see Johansson and Löfgren (1985) for details).

If there is full employment in both the short–run and the long–run, the extraction rule stated above is also socially optimal. However, if there is unemployment in the short–run, the final term in (16) must be accounted for. In Johansson (1984) it is shown that increased extraction by the state–owned firm in period 1 causes w_2 to fall. In turn, this is shown to induce private sector profit maximizing extractors to increase extraction in period 2, i.e. to reduce extraction and employment in period 1. Hence, aggregate employment in period 1, i.e. $\partial\bar{\ell}_1/\partial\ell^N_{g1}$ in (16), may rise or fall.

These effects suggest a priori that a reallocation of extraction by the state–owned firm from full–employment periods to unemployment periods may cause a rise or a fall in monetary welfare. Recall that the sum of the three first terms as well as the final term in (16) may be positive or negative.

As this analysis has demonstrated, it is far from self–evident that the presence of unemployment changes extraction rules for an exhaustible resource in a way predictable from the traditional partial equilibrium rule. According to the partial equilibrium rule discussed in Section 4.1, production of a good should be increased as long as the marginal revenue product of labor exceeds the virtual price of unemployed laborers. In light of the discussion following equation (16), such a rule of thumb may be quite misleading when applied to natural resources. This conclusion generalizes both to the case of other disequilibrium situations than the one considered in this section and to the harvest of a renewable resource. Due to considerations of space, we resist any temptations to prove these claims, but the interested reader is referred to Johansson and Löfgren (1985) for a full treatment.

6 DISTRIBUTIONAL ISSUES[8]

Most derivations of cost-benefit rules focus on efficiency considerations. Sentences like the one at the very beginning of Section 3c above, where a representative household is introduced, are therefore very common in the literature. Important exceptions are Boadway (1974), Harberger (1978), Just et al (1982), and Weisbrod (1968). The underlying idea is that since the cost-benefit rule essentially tells us whether national income has increased or not, it should be "intuitively clear" that if income indeed has increased – the aggregate budget constraint has moved outwards – the winners are able to compensate the losers.

This is true in the following limited sense. Let $x_i^{\prime}$ be a vector of commodities allocated to household i, where i = 1, ..., n, and say that there are H commodities. Moreover, let $\sum_{i=1}^{n} x_i^{\prime} = X^{\prime}$ be the aggregate bundle, while the vector $x^{\prime} = \{x_1^{\prime}, ..., x_n^{\prime}\}$ tells us how it is

[8] Note that the model in this section is not directly comparable with the above multi-sectoral model, and that the notation therefore differs slightly.

allocated over households. We can now define a reallocation of x', denoted x'', such that $\sum_{i=1}^{n} x_i'' = X'$. The winners are (potentially) able to compensate the losers in the compensation sense; i.e., x' is better than x (the initial allocation), if there is an allocation x'' with $\Sigma x_i'' = \Sigma x_i'$ and $x_i'' \succ_i x_i$ for all consumers i. (All households strictly prefer x'' to x.)

The following well-known result for the general equilibrium case can now the proved[9].

Proposition 1: (i) If x' is preferred to x in the compensation sense it must be true that $\sum_{i=1}^{n} px_i' > \sum_{i=1}^{n} px_i$, where p is a vector of H general equilibrium prices supporting x.

(ii) If $\sum_{i=1}^{n} px_i' = pX' > pX = \sum_{i=1}^{n} px_i$, and $|x_i' - x_i|$ is small for all i, there is a reallocation of x' – call it x'' – such that everyone strictly prefers x'' to x.

Since the proof is short and can be used to illuminate how things change in a disequilibrium setting we will sketch it here. To prove (i) we use the reallocation x'' such that $\Sigma x_i'' = \Sigma x_i'$, and $x_i'' \succ_i x_i$ for all i. Since x is a general equilibrium allocation supported by p it holds that $px_i'' > px_i$ for all i. Summing over i yields $\sum_{i=1}^{n} px_i'' > \sum_{i=1}^{n} px_i$. Since $\sum_{i=1}^{n} px_i'' = \sum_{i=1}^{n} px_i'$ this establishes the necessity claim.

To "prove" the sufficiency claim, (ii), we note that if $|x_i' - x_i|$ is small for all i, then

$$U_i(x_i') - U(x_i) \approx DU_i(x_i)\,(x_i' - x_i) = \lambda_i p(x_i' - x_i), \tag{17}$$

where DU_i is the gradient of the utility function, and λ_i is the marginal utility of income. In other words, the utility change can be approximated with the first order term of the Taylor series expansion. Now define x'' by:

$$x_i'' = x_i + \frac{1}{n}(X' - X), \tag{18}$$

i.e., each household is given $\frac{1}{n}$ th of the aggregate change in moving from x to x′. We now have[10]

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[9] See Varian (1984), chapter 7.

[10] Note that we are assuming that preferences can be approximated by a continuously differentiable utility function, and that we are dealing with an interior solution x_i. By shrinking the distance $|x_i' - x_i|$, the absolute error can be made smaller than any $\epsilon > 0$.

$$U_i(x_i'') - U_i(x_i) \approx \lambda_{in}^{p}(X' - X) > 0 \quad (19)$$

for all i, since $pX' > pX$ by assumption. Hence, loosely speaking, $pX' > pX$ (national income increases) is both necessary and sufficient for x' to dominate x according to the compensation criterion, provided that projects are small.The term small projects here means that they are of the same magnitude as those dealt with in the previous sections.

Let us next investigate how things are changed if there is disequilibrium. A little thought reveals that a similar proposition, where national income is evaluated at disequilibrium prices is not necessarily valid. The reason is, as we have just shown, that one has to use shadow prices to value the real effects from the projects under consideration. Evaluated at the ruling prices, markets can either be in excess demand or excess supply. These two cases are illustrated in Figure 3 below.

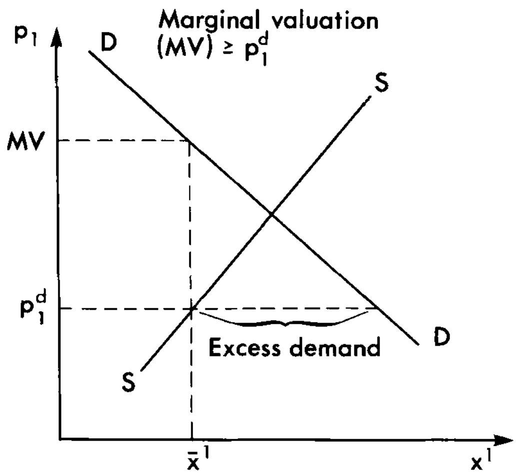

Figure 3a. Illustration of the case where price is fixed below its market–clearing level.

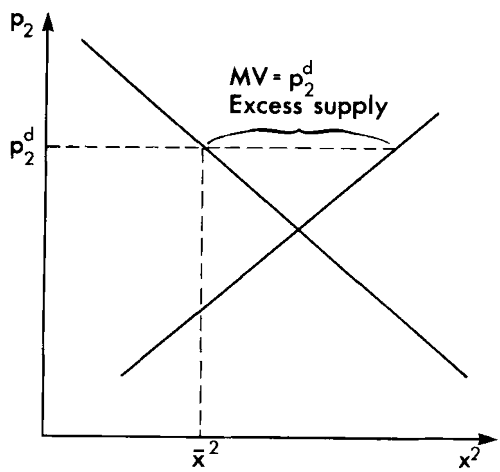

Figure 3b. Illustration of the case where price is fixed above its market–clearing level.

In the excess demand case the buyers' marginal valuation of the last unit is greater than or equal to the market price, while in the excess supply case the two entities coincide. If trade is voluntary, markets are frictionless, and all consumers are net demanders in all markets, then all consumers' marginal valuations in all markets are at least as high as the market price. In other words

$$DU_i(x_i) \geq \lambda_i p^d, \tag{20}$$

where p^d is the disequilibrium price vector. The inequality will hold with equality for components corresponding to goods in excess supply. Assume now that $p^d X' > p^d X$, then it is tempting to use the ideas in the above sketch of the proof of (ii) to try and show that this is sufficient for x' to dominate x in the compensation sense, given that $| x_i' - x_i |$ is small for all i. On face then, we would like to be able to prove that if national income (measured at disequilibrium prices) increases through the project, then it is profitable in the compensation sense.

This will, however, not work, since the fact that every component of the vector $DU_i(\cdot)$ is positive and at least as large as every component of the positive vector $\lambda_i p^d$, and $\lambda_i p^d(x_i'' - x_i) > 0$, does not imply that $DU_i(x_i'' - x_i) > 0$. In less formal language the problem arises because both the cost from a decreased supply and the benefits from an increased supply are undervalued by the disequilibrium prices. The best we have been able to do in terms of sufficient conditions is the following:

Proposition 2: If all consumer are net demanders of all goods $p^d X' \geqq p^d X$, $| x_i' - x_i |$ is small for all i, the supply of all goods in excess demand at x is not decreased at x', and the supply of at least one good in excess demand is strictly increased, then x' is strictly preferred to x in the compensation sense.

To understand this claim it suffices to note that since goods in excess demand are undervalued by the price vector and increase in supply, the reallocation principle used in "the proof" of (ii) in Proposition 1, will do the job also in this case. For a more formal argument see appendix.

The necessity proof of claim (i) of Proposition 1 will obviously not go through under disequilibrium. To see this, we note that Proposition 2 tells us that $p^d X' = p^d X$ may be sufficient for x' to strictly dominate x in the compensation sense. A simple continuity argument indicates that it may do so even if $p^d X' < p^d X$. In other words, a project can under disequilibrium conditions improve welfare in the compensation sense, even if it decreases national income measured at disequilibrium prices. The reason is, of course, that market prices underestimate the true utility gains, and this is also why shadow prices appear in the cost-benefit rules derived above. If, however, the project decreases national income at disequilibrium prices, for welfare to improve in the compensation sense, it is necessary that the project increases the supply of at least one commodity initially in excess demand (one undervalued good).

Finally, it is worth reminding the reader that the compensation criterion has its flaws, e.g., that it gives no guidance in making comparisons between Pareto efficient allocations on the same utility frontier, and that it can result in paradoxical comparisons between points on different utility frontiers. Also, if winners do in fact compensate the losers, welfare will unambiguously increase in the Pareto sense. On the other hand it is not at all clear why one should regard x' better than x merely because it is potentially possible to make everyone better off by moving to a new allocation x''.

7 CONCLUDING COMMENTS

One of the main messages that follows from the recent developments of disequilibrium cost-benefit analysis is that the partial equilibrium view of disequilibrium, which has frequently been practiced in project analysis, and which e.g., under unemployment conditions assumes that labor resources are drawn from the pool of unemployed, can be very misleading.

There are crowding out effects that mean that even if one assumes the individual supply price of unemployed resources to be zero, the total real opportunity cost of public sector employment may exceed the wage rate. For example, if total employment decreases, this may imply that profitability calculated at producer prices (the general equilibrium rule) is not sufficient to ensure that a project is socially profitable, while the incorrect partial equilibrium rule does not even require profitability measured at producer prices.

This paper in particular shows that the intertemporal aspects may also be important. Under rational expectations a public project planned for a future full employment situation may inflict extra social costs today, through a lower future price which induces a lower private investment activity today, causing a decrease in today's employment.

Intertemporal considerations are also shown to modify some of the more "counterintuitive" conclusions arrived at by earlier investigators of "atemporal" models. For example, a decreased total employment is no longer necessary for a public project in nontradeables to improve welfare under Keynesian unemployment. The reasons are that crowding out effects are smaller in an intertemporal setting, and that, interestingly enough, existing crowding out effects stimulate investment.

The public projects that are discussed in this paper are financed by nondistortionary lump-sum taxes. This has, of course, affected the exact shape of the cost-benefit rules. Under a more general tax system the rules would contain terms measuring the deadweight losses from taxation. There is, however, an interesting exception worth pointing out. A variable tax on the wage rate works like a lump-sum tax when the household is rationed in the labor market. What normally distinguishes a lump-sum tax from a tax on a good or a factor is that it has an income effect but no substitution effect. However, if the household is rationed in the labor market, a change in the after-tax wage, like a lump-sum tax, will have only an income effect on the demand for unrationed goods.

Needless to say, the disequilibrium paradigm can be used to study the direction of tax reform, optimal taxation, and to develop optimal shadow pricing rules[11], given restrictions on the shape of the tax system. For example, optimal shadow prices are obtained by maximizing the social indirect utility function with respect to the project parameters. To see this, if prices are market clearing, the gradient of this function with respect to prices vanishes like equation (6) tells us. These developments, however, must be relegated to another paper. Note, however, that the fact that tomorrow's policy parameters affect today's decisions imply that optimal policy rules may be time inconsistent. For example, if we evaluate the relief works mentioned in Section 4.1 in the full employment period we will be inclined to use the time consistent, but inoptimal general equilibrium evaluation rule, and, hence, wrongly neglect any extra costs in the preceeding unemployment period. See Kydland and Prescott (1977) for details.

The distributional aspects of cost-benefit analysis are often dominated by more straightforward efficiency considerations. In this paper an attempt is made to derive conditions for one state to dominate another in terms of the Kaldorian welfare criterion. It turns out that the resulting conditions are slightly more complex than under equilibrium, but that they can be expressed in ruling nonmarket clearing prices, and that they lend themselves to simple intuitive explanations. The main message is that the likelihood of welfare improvements is enhanced if the projects generate net supplies of goods in excess demand undervalued goods. In particular, profitable compensation may be possible, even if the project in question decreases national income measured at ruling disequilibrium prices, provided that it increases net supplies of goods in excess demand sufficiently.

To sum up: Given the non-negligible first order induced effects of even small projects under disequilibrium conditions, it is of first order importance to find out their magnitude in relation to more obvious direct effects. Today's macroeconometric model building paired with the speed of todays computer's indicate that simulation techniques can be used to accomplish this task. A recent attempt by Fourgeaud et al (1986), where shadow prices are estimated numerically for the French economy illustrates the feasibility of the approach.

APPENDIX: PROOFS OF CLAIMS IN THE MAIN TEXT

1 Proof that $\partial I/\partial\bar{x}_1 < 0$

In order to derive the sign of $\partial I_1/\partial\bar{x}_1$ for a firm facing a sales constraint in period 1, the

[11] See e.g. Guesnerie (1978), and Hammond (1986) for analysis of tax reform in a general equilibrium context. An interesting dual approach to the second best problem is pursued in Guesnerie and Roberts (1984), who investigate how quantity rationing can be used as an instrument to achieve optimality. Optimal policy rules and regime switching is analyzed in Cuddington et al (1985) and Marchand et al (1985).

profit maximization problem of the firm is written as:

$$\underset{\ell}{\text{Max}}\ \hat{\Pi} = p_1\bar{x}_1 - w_1\ell_1 + p_2H[\ell_2, F(\ell_1) - \bar{x}_1] - w_2\ell_2 - p_2I_2 + \phi(I_2, \odot) \tag{A.1}$$

where we have used the fact that $\bar{x}_1 = y_1 - I_1$ and $y_1 = F(\ell_1)$, i.e. $I_1 = F(\ell_1) - \bar{x}_1$, and a superscript i referring to the kind of commodity produced by the considered firm is suppressed in order to simplify the exposition. Throughout both the first-period and the second-period production functions are assumed to be strongly concave.

The necessary (and sufficient) conditions for profit maximization are:

$$- w_1 + p_2H_IF_\ell = 0 \tag{A.2}$$

$$- w_2 + p_2H_\ell = 0$$

$$- p_2 + \phi_I = 0$$

where subscripts ℓ and I refer to partial derivatives with respect to ℓ and I, respectively, and F_ℓ, H_ℓ, H_I, $\phi_I > 0$ by assumption.

Tedious but straightforward calculations, using (A.2), show that:

$$dy_1/d\bar{x}_1 = (A - p_2^2H_IF_{\ell\ell}H_{\ell\ell})/A < 1 \tag{A.3}$$

where $A = p_2^2(H_{II}H_{\ell\ell}F_\ell^2 - H_{\ell I}^2F_\ell^2 + H_IF_{\ell\ell}H_{\ell\ell}) > 0$, and subscripts ℓI, etc, refer to cross-derivatives. A is positive since, for a strictly concave production function, $H_{II}H_{\ell\ell} - H_{\ell I}^2 > 0$ and F_{ii}, $H_{ii} < 0$ for $i = \ell$, I.

Next, let us consider the following partial derivatives of the profit function (11):

$$\partial\hat{\Pi}/\partial p_1 = \bar{x}_1 = y_1 - I_1 \tag{A.4}$$

$$\partial^2\hat{\Pi}/\partial p_1\partial\bar{x}_1 = 1 = (\partial y_1/\partial\bar{x}_1) - (\partial I_1/\partial\bar{x}_1)$$

Combining (A.3) and (A.4), one finds that $\partial I/\partial\bar{x}_1 < 0$.

2 Proof of Proposition 2

To prove Proposition 2 in Section 7, we first note that if one person can be made strictly better off than in x by redistribution of a vector x', and no one worse-off, we can in a non-satiated solution make everybody better off by redistributing some of the gain to the non-gainers. In other words, if we can prove that there is a redistribution of x′ – call it x" – such that at least one household is strictly better off and no household worse off, we are through.

The change in utility in moving from x_i to x_i'' can if $| x_i'' - x_i |$ is small enough be approximated to any degree by the first order terms in the Taylor series expression of the utility function, i.e., $U(x_i'') - U(x_i) \approx DU(x_i)(x_i'' - x_i)$

If we partition the vector of goods into goods in excess demand $(x_i'' - x_i)_e$ and goods in excess supply $(x_i'' - x_i)_s$, with prices p^e and p^s, respectively, we can rewrite the above expression in the following manner:

$$U(x_i'') - U(x_i) \approx DU(x_i)(x_i'' - x_i) = D^eU(x_i)(x_i'' - x_i)_e + \lambda_i p^s (x_i'' - x_i)_s \qquad (A.5)$$

where $D^eU_i(x_i)$ is a vector of marginal utilities corresponding to the goods in excess demand. If x_i'' is defined as

$$x_i'' = x_i + \frac{1}{n}[(X_e', X_s') - (X_e, X_s)] = x_i + \frac{1}{n}(X' - X) \qquad (A.6)$$

we have

$$U(x_i'') - U(x_i) \approx \frac{D^eU(x_i)}{n}(X_e' - X_e) + \frac{\lambda_i\ p^s}{n}(X_s' - X_s) \qquad (A.7)$$

Since $X_e' - X_e \geqq 0$ by assumption, and $D^eU(x_i) \geqq \lambda_i p^e$ it follows that

$$\frac{D^eU(x_i)}{n}(X_e' - X_e) + \frac{\lambda_i\ p^s}{n}(X_s' - X_s) \geqq \frac{\lambda_i\ p^d}{n}(X' - X) \geqq 0 \qquad (A.8)$$

for all i, with strict inequality for at least one i in the first inequality, since the supply of at least one good in excess demand increases, and at least one household must be rationed in an excess demand market $(\frac{\partial U}{\partial x_i^h} > \lambda_i\ p_h)$.

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